First Volume

Formula: 1. General Cross Section:

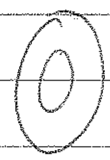
$$V = \int_a^b A(x)\,dx$$

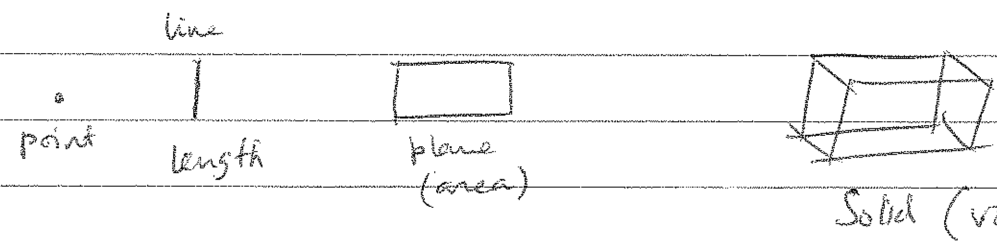
9

2. Solid of revolution: $V = \pi \int_a^b \text{Radius}^2\,dx$ (disk)

3. Washer method: $V = \pi \int_a^b (\text{outer radius}^2 - \text{Inner}^2)\,d$

(washer)





2

← differentiation —— integration →

Webpage:

bankimmath.weebly.com/teaching.html.

Office hours: Wednesday 10:15 - 12. C 227

4 ©

$[\sin(x)]^{-1} = \frac{1}{\sin x}$

Trig Table:

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin(x)	$\sqrt{0}/2$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
	$\sqrt{4}/$	$\sqrt{3}/$	$\sqrt{2}/$	$\sqrt{1}/2$	$\sqrt{0}/2$

4 c

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$\sin(x)$	$\sqrt{0}/2$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos(x)$	$\sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2$	$\sqrt{0}/2$
$\tan(x)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undef.

2

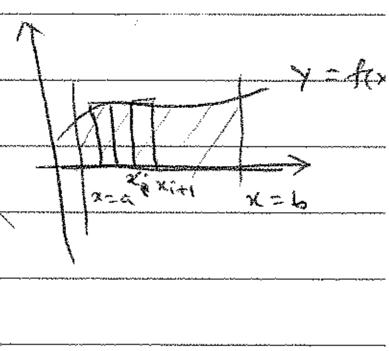
Nov. 5.1:

These are few application of definite integration. Compute area between curves, volume of solids, or work done by a force.

→ Integration is a tool to calculate the area between curves.

→ Finding the area bounded by a given curve $y = f(x)$, the lines $x = a$ & $x = b$ & x-axis ($y > 0$)

Consider small rectangular strips of equal width. Δx



Partitioning: $(x_0, x_1, \dots x_n)$ $\quad \Delta x = \frac{b-a}{n}$

[a, b] $\quad x_0 = a$, $x_n = b$.

$$S \sim \sum_{i=1}^{n} \overbrace{f(t_i)\,\Delta x}^{A_i} \quad \text{for } x_{i-1} \le t_i \le x_i$$

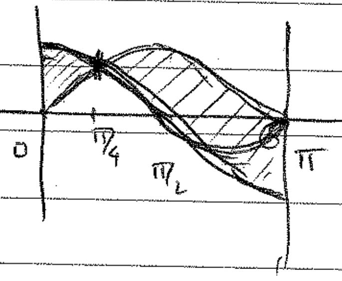
approximation of the area.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i = \int_a^b f(x)\,dx$$

instead $[f(x) - g(x)]$

(2nd) 6/ Friday -1

Example: Find the area between the curves $y = \sin x$ & $y = \cos x$ for $0 \le x \le \pi$



$\sin x = \cos x = \sin(\pi/2 - x)$

[do not write $\tan x = 1$]

$2x = \pi/2$

$\Rightarrow x = \pi/4$.

The ~~integral~~ area has two parts:

$$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\, dx = \Big[\sin x + \cos x\Big]_0^{\pi/4}$$
$$= \sqrt{2} - 1$$

$$A_2 = \int_{\pi/4}^{\pi} [\sin(x) - \cos x]\, dx = -\cos x - \sin x \Big]_{\pi/4}^{\pi}$$
$$= 1 + \sqrt{2}$$

The total area $= (\sqrt{2} - 1) + (1 + \sqrt{2}) = \boxed{2\sqrt{2}}$

"Signed area" $\tilde{A} = \int_0^{\pi} [\cos(x) - \sin(x)]\, dx = (\sqrt{2} - 1) - (1 + \sqrt{2})$
$= -2 = A_1 - A_2$

Tricks: 1. sketch the curve.
2. Sketch an element $dA = f(x)\, dx$.
3. Sketch the limits of integration.

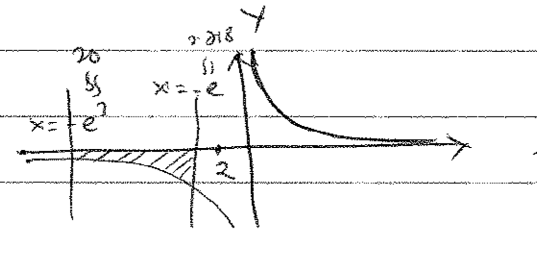
Exm 1: 5/

Find the area between x-axis, the curve $y = 1/x$ and the lines $x = -e^3$ & $x = -e$ ($e = 2.718\ldots$)

Soln: For $-e^3 \le x \le -e$

$e^3 > e$
$\Downarrow$

$f(x) = 0$



Exm 1: 5/ ©

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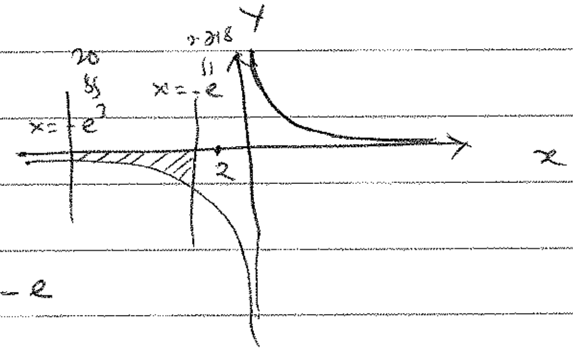


$e^3 > e$

$\Downarrow$

$-e^3 < -e$

$f(x) = 0$

$g(x) = 1/x$

$$\text{Area} = \int_{-e^3}^{-e} (0 - 1/x)\,dx = -\ln|x|\Big]_{-e^3}^{-e}$$

$$= -(\ln e - \ln e^3)$$

$$= -(1-3) = \boxed{2 \text{ sq. units.}}$$

go to p-6

Q.H.W: A region containing the origin is cut out by the curves $y = 1/\sqrt{x}$, $y = -1/\sqrt{x}$, $y = 1/\sqrt{-x}$ & $y = -1/\sqrt{-x}$ and the lines $x = \pm 4$, $y = \pm 4$. ~~Find the area of the region.~~

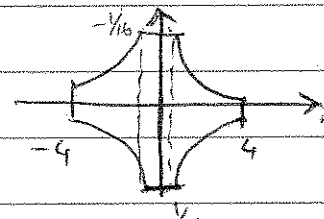
$\sqrt{x} = \frac{1}{4}$

$x = \frac{1}{16}$

Hint:

$y = 4$. $4 = \pm\frac{1}{\sqrt{-x}}$

$-x = \frac{1}{16}$

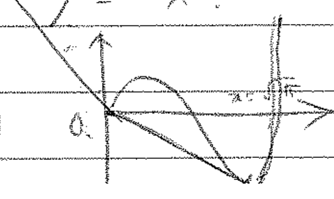


$\frac{1}{8} \times 8 = 1$

(or formulate the integration)

Q.H.W: Find the area of the leftmost region enclosed below the curve $y = x\cos(x^2)$ and above the line $y = -x$.

Hint: $A = \int^{\sqrt{\pi}} [x\cos(x^2) + x]\,dx$ $(= \pi/2)$



Friday-2

Section 5.2 Volume. $\frac{7}{} \to 7a$ 3

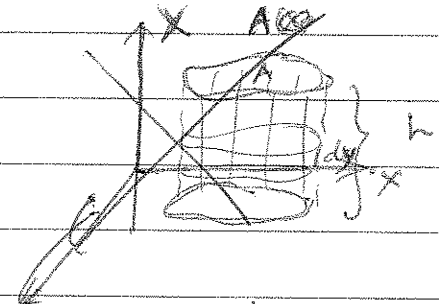
Circular cylinder: $\pi r^2 \times h$

Rectangle: $l\,w\,h = A \times h$, Sphere: $\frac{4}{3}\pi r^3$.

Slice Method: $\neq \pi r^2 \cdot (r\ 4/_3)$

This is analogous to the area approximation ~~of~~ by slices:

Take a surface, consider the area underneath the surface.
We really have volume.

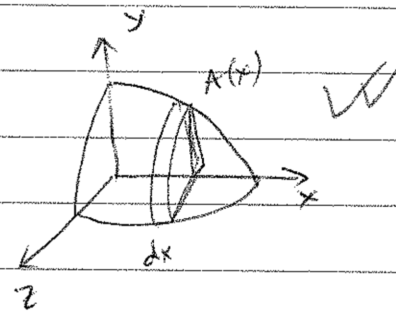


Volume = Area × height.

$$V \approx \lim_{n\to\infty} \sum_{k=1}^{n} A(x_k) \times \Delta x$$

$$= \int_a^b A(x)\,dx$$

$dV = A(x)\,dx$

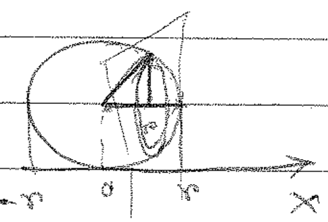


Exm 1

* Find the volume of a ball of radius r.

The area of the slice of P_x $= \pi\left(\sqrt{r^2-x^2}\right)^2$

$= \pi\,(r^2-x^2)$



P_0 is the plane pass through the center.

So, the volume $= \displaystyle\int_{-r}^{r} A(x)\,dx = \int_{-r}^{r} \pi(r^2-x^2)\,dx$

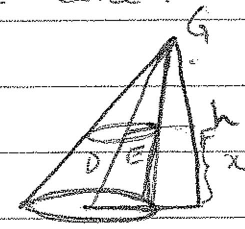
$$= \pi\left(r^2x - \frac{x^3}{3}\right)\Bigg|_{-r}^{r}$$

$= \frac{4}{3}\pi r^3$

F4 7a 8

Q.H.W: Find the volume of the oblique cone:

Next week. $\left(\frac{1}{3}\pi r^2 h\right)$ Give answer!



5

Solids of revolution :- (Disk method)

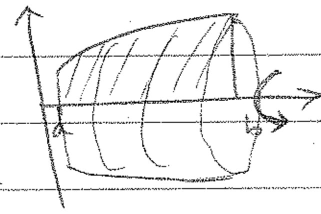
GE

$\left(\frac{DE}{AB} = \frac{GE}{GB}\right.$

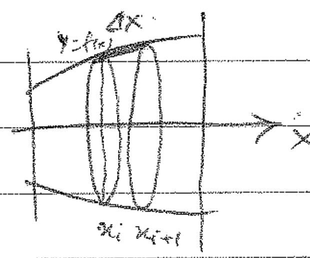
$= \frac{GB}{Ge}$

simil tria

Volume of that solid, generated by revolving the region around a line. (x-axis)



Area $= \pi[f(x)]^2$



ΔV_i = volume slice

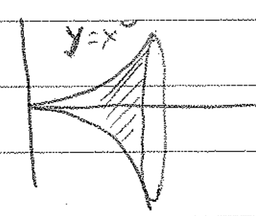
$= \pi(f(x_i))^2 \Delta x$.

$$V = \lim_{n\to\infty} \sum \pi f(x_i)^2 \Delta x = \int_a^b \pi f(x)^2 dx.$$

Ex3: The region under the graph of x^2 on $(0,1)$ is revolved about the x-axis. Sketch and compute the volume

$$V = \int_0^1 \pi (x^2)^2 dx$$

$$= \pi \frac{1}{5}.$$



Friday-2

7 → 7a

Section 5.2 Volume.

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Rectangle: $l\,w\,h = A \times h$, Sphere: $\frac{4}{3}\pi r^3$.

$\neq \pi r^2 (r\,4/3)$

Slice Method:

This is analogous to the area approximation ~~by~~ slices:

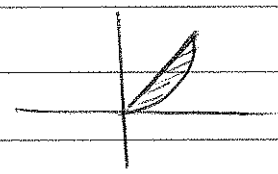
Take a surface, consider the area underneath the surface

we really have volume.

X A

Last 11

Washer method Find the volume of the solid bdd by the lines $y = x^2$ & $y = 2x$ in the first quadrant and revol around y axis.



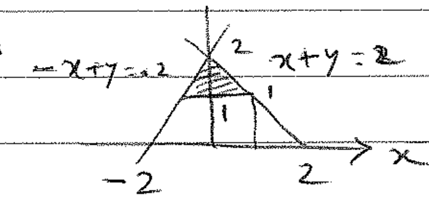
7

$$x^2 = 2x \Rightarrow x(x-2) = 0$$
$$x = 0, 2.$$

$$V = \int_0^4 \pi \left[x_1^2 - x_2^2 \right] dy$$
$$= \int_0^4 \pi \left[y - y^2/4 \right] dy = 8\pi/3 \text{ unit}^3$$

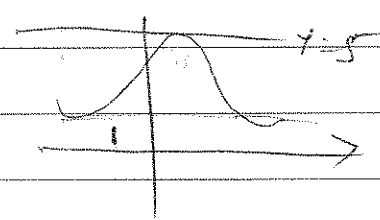
10

* Rotate the shaded region around x-axis



$V = 2 \cdot$ volume by rotating R.
$$= 2\pi \int_0^1 \left[(-x+2)^2 - 1 \right] dx$$
$$= 2\pi \int_0^1 (x^2 - 4x + 4 - 1)\, dx = \frac{8}{3}\pi.$$

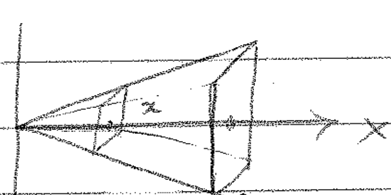
* Find the volume of the solid generated by rotating the region betw the line $y = 5$ & $y = 2\sin x + 3$ for $-\pi/2 \le x \le 3\pi/2$ about $y = 5$



$$V = \int_{-\pi/2}^{3\pi/2} \pi \left(5 - (2\sin x + 3)\right)^2 dx$$
$$= 12\pi^2$$

Slice F3 0 7a 4

Exm 2: Find the volume of a pyramid whose base is a squ with side L and height h.

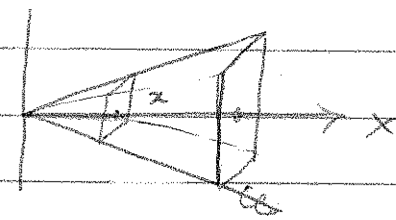


$$V = \int^h A(x)\, dx = \frac{L^2 h}{3}$$

$A(x) = s^2 = \left(\frac{L}{h} x\right)^2$

Slice F3 0 ~~7a~~ 4

Exm 2: Find the volume of a pyramid whose base is a square with side L and height h.

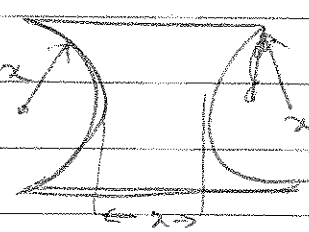


$$V = \int_0^h A(x)\,dx = \frac{L^2 h}{3}$$

$A(x) = S^2 = \left(\frac{L}{h}x\right)^2$

Slice method: Ex.

Cross Section



Solid beam with cross section
Find cross sectional area $A(x)$
$0 \le x \le 5$, and the volume.

$$A(x) = 6x^2 - \pi x^2$$

$$V = \int_0^5 (6-\pi)\, x^2\, dx = 250 - \frac{125}{3}\pi$$

Finally. Example 9 if time permits.