Tut 4

Gerschgorin Theorem :-

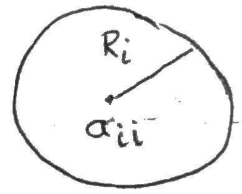
Let, A be an $n \times n$ matrix and

$$R_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \quad \text{for ith row}, \ i = 1, 2, \ldots, n.$$

Consider the disc :

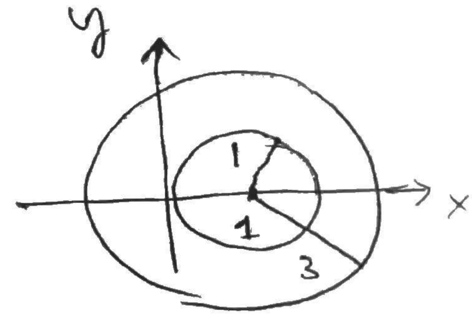
$$D_i(a_{ii}, R_i) = \left\{ |z - a_{ii}| \leq R_i \right\} .$$

Every eigenvalue of A lies within at least one of the Gerschgorin disk D_i.



Exm :

$$A = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}$$



$D_1(1,3) = |z-1| \leq 3$

$D_2(1,1) = |z-1| \leq 1$

Two eigen-values of A lie in either D_1 or D_2

i.e. $-2 \leq \lambda_1, \lambda_2 \leq 4$ if real λ_i.

So, $\underline{\rho(A) \leq 4}$.

3.

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$M_J = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Disks:

$D_1(0, \frac{1}{2})$, $D_2(0,1)$, $D_3(0,1)$, $D_4(0, \frac{1}{2})$

$\therefore -1 \leq \lambda_i \leq 1 \Rightarrow \rho(M_J) \leq 1$

Check: $\lambda = 1$ not an eigenvalue

$$M_{GS} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} \\ 0 & \frac{1}{16} & \frac{1}{8} & \frac{1}{4} \end{pmatrix}$$

Disks

$D_1(0, \frac{1}{2})$ $D_2(\frac{1}{4}, \frac{1}{2})$

$D_3(\frac{1}{4}, \frac{3}{8})$, $D_4(\frac{1}{4}, \frac{3}{16})$

$\therefore 0 \leq |\lambda_i| \leq \frac{3}{4}$.

$\therefore \rho(M_{GS}) \leq \frac{3}{4}$