5

$$\lim_{x\to\infty} \frac{\ln\sqrt{x}}{\log_4 x} = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}{\frac{1}{\ln 4}\cdot 1/x} = \frac{\ln 4}{2}.$$

What is $\lim_{x\to 0} \frac{x^2+1}{x}$?

Q.H.W : $\lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x^2}\right)$ $(\infty-\infty)$

p-483

101 $\int \tan x \ln(\cos x)\, dx$ Let, $\ln(\cos x) = u$

$-\frac{\sin x}{\cos x} dx = du$

$= -\int u\, du$

$= -\frac{1}{2}u^2 + c = -\frac{1}{2}[\ln(\cos x)]^2 + c$.

1

7.1 Integration by Parts :-

$$\frac{d}{dx}(f g) = \frac{d}{dx}f \cdot g + f\cdot\frac{d}{dx}g.$$

$$\Rightarrow \int \frac{d}{dx}(fg)\,dx = \int f'g\,dx + \int fg'\,dx$$

$$\Rightarrow fg = \int f'g\,dx + \int fg'\,dx.$$

$$\Rightarrow \int fg'\,dx = fg - \int f'g\,dx$$

Or $\int u\,dv = uv - \int v\,du.$

Exm: $\int x^2 \sin x\,dx = x^2 \int \sin x\,dx + \cancel{\ldots}$

$+\int 2x \cos x\,dx$

$= -x^2\cos x + 2\int x\cos x\,dx.$

$= -x^2\cos x + 2\,x.\sin x. - 2\int \sin x\,dx + c$

$= -x^2\cos x + 2x\sin x + 2\cos x + c.$

Exm $\int \ln x\,dx. = \ln x.\int dx - \int \frac{1}{x}\left(\int dx\right) dx$

$= x\ln x - \int \frac{1}{x}.\,x\,dx + c$

$= x\ln x - x + c$

Exm: $I = \int e^x \cos x\,dx.$ $\quad u = e^x,\ dv = \cos x\,dx.$

$v = \sin x$

$= +e^x \sin x - \int e^x \sin x\,dx.$

$= e^x \sin x + \cancel{e^x \cos x\,dx} \mp \int e^x \cos x\,dx$

$= e^x \sin x + e^x \cos x - I + 2c.$

$\therefore\ 2I = e^x \sin x + e^x \cos x. + 2c.$

$\therefore\ I = \frac{1}{2} e^x (\sin x + \cos x) + c$

#

$$\int_1^e \sin(\ln x)\,dx \qquad \text{Let, } u = \ln x$$

$$du = \frac{1}{x}\,dx.$$

$$= \int_0^1 \sin u \cdot e^u\,du$$

$$= \int_0^1 e^u \sin u\,du = \frac{1}{2} e^u(\sin u - \cos u)\Big]_0^1$$

$$= \frac{e}{2}\left[\sin 1 - \cos 1 + \frac{1}{e}\right]$$

#

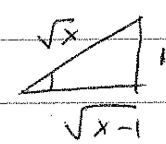
$$\int \operatorname{Cosec}^{-1}(\sqrt{x})\,dx.$$

$$\text{Let, } u = \operatorname{Cosec}^{-1}(\sqrt{x}).$$

$$= \int u\,dx. \qquad \sqrt{x} = \operatorname{Cosec} u$$

$$= ux - \int x\,du.$$

$$= x\operatorname{Cosec}^{-1}(\sqrt{x}) - \int \operatorname{Cosec}^2 u\,du$$



$$= x\operatorname{Cosec}^{-1}(\sqrt{x}) + \operatorname{Cot}(u) + c.$$

$$= x\operatorname{cosec}^{-1}(\sqrt{x}) + \operatorname{Cot}(\operatorname{Cosec}^{-1}(\sqrt{x})) + c.$$

$$= x\operatorname{Cosec}^{-1}(\sqrt{x}) + \sqrt{x-1} + c.$$

#

$$\int x\cos^{-1}x\,dx \qquad u = \cos^{-1}x \qquad dv = x\,dx$$

$$du = -\frac{1}{\sqrt{1-x^2}}\,dx \qquad v = x^2/2$$

$$= \frac{x^2}{2}\cos^{-1}x + \frac{1}{2}\int \frac{x^2\,dx}{\sqrt{1-x^2}}$$

$$\text{Let } x = \cos u$$

$$= \frac{x^2}{2}\cos^{-1}x \mp \frac{1}{2}\int \frac{\cos^2 u\,\cancel{\sin u}\,du}{\cancel{\sin u}} \qquad dx = -\sin u\,du$$

$$= \frac{x^2}{2}\cos^{-1}x - \frac{1}{4}\int (1+\cos 2u)\,du = \frac{x^2}{2}\cos^{-1}x - \frac{1}{4}\left[u + \frac{\sin 2u}{2}\right] + c$$

$$= \frac{x^2}{2}\cos^{-1}x - \frac{1}{4}\cos^{-1}x - \frac{1}{4}x\sqrt{1-x^2} + c$$

(4)

$\int \tan^{-1}x \, dx.$ $\quad u = \tan^{-1}x. \quad dv = dx.$

$du = \dfrac{dx}{1+x^2} \quad v = x.$

Important

$$\int U(x)\,W(x)\,dx = W(x)\int U(x)\,dx - \int \frac{dW}{dx}\left[\int U(x)\,dx\right]dx.$$

$$= \tan^{-1}x \int 1\,dx - \int \frac{1}{1+x^2}\,x\,dx.$$

$$= x\tan^{-1}x - \frac{1}{2}\int \frac{du}{1+u} \qquad x^2 = u, \quad 2x\,dx = du$$

$$= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + c$$

$\int 13x\, 3^x dx = 13x\int 3^x dx - 13\int \frac{3^x}{\ln 3}\,dx$

$$= 13x\,\frac{3^x}{\ln 3} - 13\,\frac{1}{(\ln 3)^2}\,3^x + c.$$

$I = \int \sin(\ln x)\,.\,dx. = \sin(\ln x)\int dx - \int \frac{\cos(\ln x)}{\cancel{x}}\,.\,\cancel{x}\,dx$

$$= x\sin(\ln x) - \cos(\ln x)\int dx - \int \sin(\ln x)\,dx.$$

$$\Rightarrow \quad 2I = x\sin(\ln x) - x\cos(\ln x)$$

$$\therefore \quad I = \frac{1}{2}\Big(\sin(\ln x) - \cos(\ln x)\Big)x + c$$

$\cancel{6}$ (5)

39

$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\, d\theta .$$

Let, $x = \theta^2$

$dx = 2\theta\, d\theta$

$$= \frac{1}{2}\int_{\pi/2}^{\pi} x \cos x \, dx .$$

$$= \frac{1}{2}\left[x \int \cos x\, dx\right]_{\pi/2}^{\pi} - \frac{1}{2}\int_{\pi/2}^{\pi} \sin x\, dx .$$

$$= \frac{1}{2}\left[x \sin x\right]_{\pi/2}^{\pi} + \frac{1}{2} \cos x\Big]_{\pi/2}^{\pi} = -\frac{1}{2}\frac{\pi}{2} + \frac{1}{2}.\left[-1\right]$$

$$= -\pi/4 - 1/2$$

53 $I_n = \int \tan^n x\, dx = \int \tan^{n-2} x\,(\sec^2 x - 1)\, dx$

$$= \tan^{n-2} x \int \sec^2 x\, dx - \int (n-2)\tan^{n-3} x\ \sec^2 x . \tan x\, dx$$

$$- \int \tan^{n-2} x\ dx$$

$$= \tan^{n-1} x - (n-2)\int \tan^{n-2} x\,(1 + \tan^2 x)\, dx$$

$$- I_{n-2}$$

$$= \tan^{n-1} x - (n-2)\, I_{n-2} - (n-2)\int \tan^n x\, dx - I_{n-2}$$

$$\Rightarrow \quad (n-1)\, I_n = \tan^{n-1} x - (n-1)\, I_{n-2}$$