1.

Work

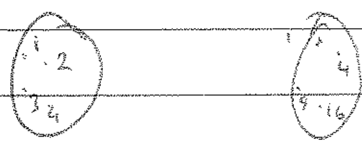
Const. force — (Easy) — $(F.d)$

Variable force — (Hooke's law, gravity with cable weight) — $\int_a^b f(x)\,dx$

Section 6.1 **Inverse functions:**

(Properties of few inverse fn)

(f, g)



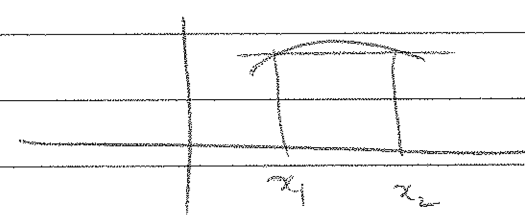
$f(n) = n^2$

f is 1-1 $\Rightarrow$ Not all fn posses inverses!

$\Rightarrow$ A fn is 1-1 if $f(x_1) \neq f(x_2) \Leftarrow x_1 \neq x_2$

* $f(x_1) = f(x_2)$

but $x_1 \neq x_2$



Horizontal line intersects the graph only once! Not 1-1

Ex. $f(x) = x^2$ ✗ $x \in \mathbb{R}$.

$f(x) = x^3$. ✓

$f'(0)$ f^{-1}

Inverse fn.:- $f: A \to B$ is 1-1. Then, its inverse fn f^{-1} has domain B and range A & is defnd by

$f^{-1}: B \to A$ $\quad f^{-1}(y) = x \iff f(x) = y$ for $y \in B$.

$$\Longrightarrow \quad f^{-1}(x) \neq \frac{1}{f(x)} = [f(x)]^{-1}$$

$\Rightarrow$ $f(f^{-1}(x)) = x$ for every $x \in B$
& $f^{-1}(f(x)) = x$ for every $x \in A$. || Cancellation eqn

Exm: $f(x) = x^5 - 1$ Find f^{-1} — Any odd powered polyn is 1-1

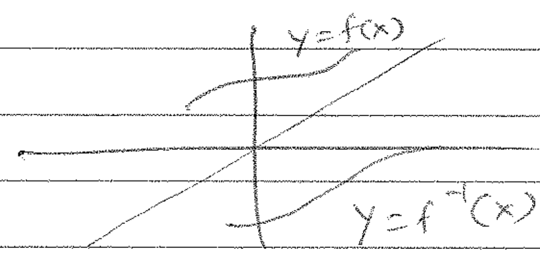
$y = x^5 - 1 \Rightarrow x^5 = y + 1$

$\Rightarrow x = (y+1)^{1/5}$

$\Rightarrow \underline{\underline{f^{-1}(x) = (x+1)^{1/5}}}$

QHW: $f(x) = \frac{x+1}{x+2}$ Find f^{-1}, domain & range.

Draw graph: The graph of f^{-1} is obtained by reflecting the graph of f about $y = x$.



Calculus of Inverse fn.: If f is cont. & 1-1, f^{-1} is also cont.

Derivative :- If f is 1-1 differentiable fn with inverse fn f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse fn is diff ~~and~~ at a &

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Pf: By defn.

$$(f^{-1})'(a) = \lim_{x \to a} \frac{f^{-1}(x) - f^{-1}(a)}{x - a}$$

Let, $f(b) = a \Rightarrow f^{-1}(a) = b$

& $x = f(y) \Rightarrow y = f^{-1}(x)$

$$\Rightarrow (f^{-1})'(a) = \lim_{x \to a} \frac{y - b}{x - a}$$

f^{-1} is cont. So, $f^{-1}(x) \to f^{-1}$ $y \to b$.

$$= \lim_{y \to b} \frac{y - b}{f(y) - f(b)}$$

$$= \lim_{y \to b} \frac{1}{\frac{f(y) - f(b)}{y - b}}$$

$$= \frac{1}{f'(b)} = \frac{1}{f'(f^{-1}(a))}.$$

Generally, $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}.$$

Ex: $f(x) = 2 + \sin x$. Find $(f^{-1})'(2)$.

$f'(x) = \cos x$. $f(0) = 2$

$f^{-1}(2) = 0$

$f'(f^{-1}(2)) = f'(0) = 1 \neq 0$, $(f^{-1})'(2) = \dfrac{1}{f'(0)} = \dfrac{1}{1} = 1$

Inverse fn :- $f: A \to B$ is 1-1. Then, its inverse fn f^{-1} has domain B and range A & is defined by

~~$f^{-1}: B \to A$~~ $f^{-1}(y) = x \Leftrightarrow f(x) = y$. for $y \in B$.

$$\Longrightarrow f^{-1}(x) \neq \frac{1}{f(x)} = [f(x)]^{-1}$$