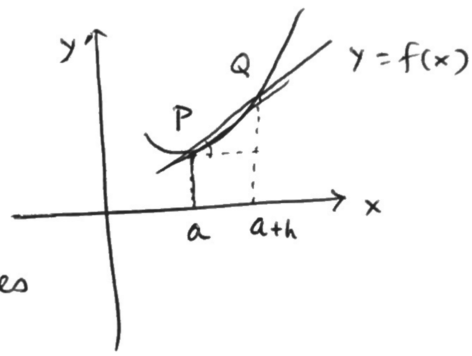
② Differentiation :-



Slope of the Secant line

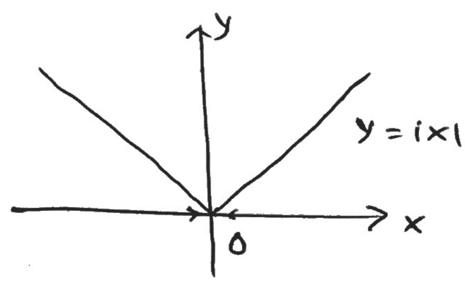
$$= \frac{f(a+h) - f(a)}{h}$$

As $Q \to P$, the secant line approaches to the tangent line at P.

$$\text{Slope of the tangent} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

If the limit exists, it is called the derivative of f at a; and denoted by $f'(a)$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$



Exm 1. $f(x) = |x|$

$$\lim_{h\to 0+} \frac{f(h) - f(0)}{h} = \lim_{h\to 0+} \frac{h}{h} = 1$$

$$\lim_{h\to 0-} \frac{f(h) - f(0)}{h} = \lim_{h\to 0-} \frac{-h}{h} = -1$$

f is not derivable at $x = 0$.

f is diff. at a, then, f is cont. at a.

$$f(x) - f(a) = \frac{f(x) - f(a)}{(x-a)} \cdot (x-a)$$

$$\Rightarrow \lim_{x\to a} (f(x) - f(a)) = f'(a) \cdot \lim_{x\to a} (x-a)$$

$$= 0.$$

$$\Rightarrow \lim_{x\to a} f(x) = f(a)$$

| Converse not true (Exm 1)

Right hand derivative

$$f'_+(a) = \lim_{h \to 0+} \frac{f(a+h) - f(a)}{h}$$

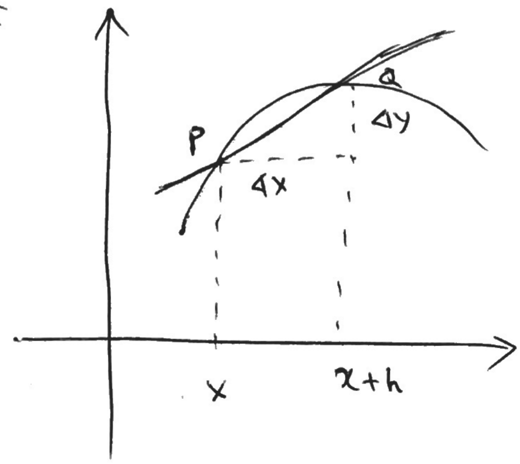
Left hand :

$$f'_-(a) = \lim_{h \to 0-} \frac{f(a+h) - f(a)}{h}$$

~~Rules of derivative~~ : ~~Skip.~~

$$\cancel{\frac{d}{dx}(f+g) = f' + g'}$$

Leibnitz Notation :-



$$\text{Slope of } PQ = \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x).$$

$\frac{d}{dx}(f(x))$: derivative of f at x.

Rules of derivative : (skip)

addition, subtraction, division, . . .

chain rule -

Mean Value Theorems :-

Rolle's theorem :- Let, $f : [a,b] \to \mathbb{R}$ be a function.

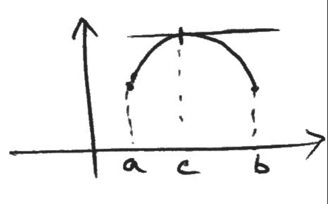
i) f is continuous on $[a,b]$

ii) f is differentiable in (a,b)

iii) $f(a) = f(b)$.

Then, $\exists$ a $c \in (a,b)$ ~~st~~ such that

$$f'(c) = 0$$



Remark

1. The conditions are suff., not necessary.

$$f(x) = |x| + |x-1| , \quad x \in [-1, 2]$$

$$= \begin{cases} 1-2x, & -1 \le x \le 0 \\ 1, & 0 \le x \le 1 \\ 2x-1, & 1 < x \le 2. \end{cases}$$

$f(-1) = f(2)$. $\quad f'(x) = 0$ in $(0,1)$

f is not derivable at $0, 1$.

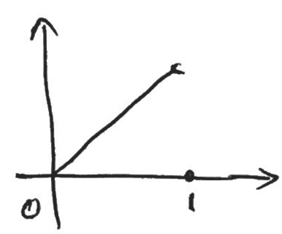
2. $f(x) = |x| , \quad x \in [-1, 1]$

f is cont, f is not diff. at $x = 0$.

$$f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

Diff. is necessary in Rolle's thm.

3. $$f(x) = \begin{cases} x, & \cancel{\text{if}}\ 0 \le x < 1 \\ 0, & x = 1 \end{cases}$$



f is not cont. at $x = 1$.

ii) f is derivable in $(0,1)$.

iii) $f(0) = f(1) = 0$.

No point c s.t. $\underline{f'(c) = 0}$.

Continuity is necc. in Rolle's theorem.