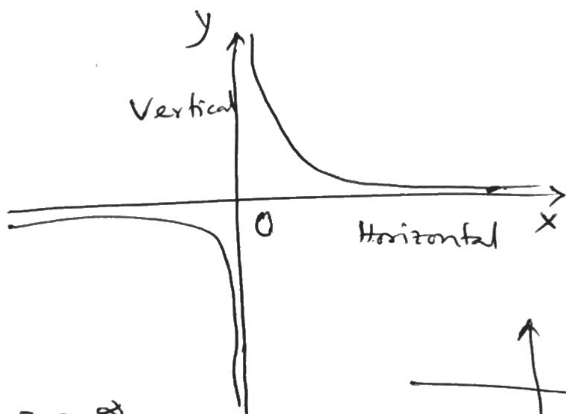
\# Let, f and g be functions, continuous on $[a,b]$, diff. on (a,b) and $f(a) = f(b) = 0$. Prove that $\exists\, c \in (a,b)$ s.t.

$$g'(c)\,f(c) + f'(c) = 0.$$

$\Rightarrow$ Apply Rolle's thm to $\underline{F(x) = f(x)\,e^{g(x)}}$.

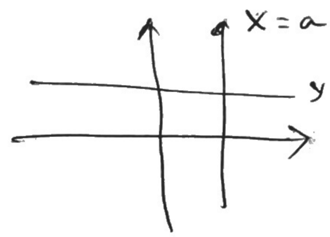
Asymptotes :

$$y = \frac{1}{x}, \quad x \neq 0$$



$$\lim_{x \to \pm\infty} \frac{1}{x} = 0$$

$$\lim_{x \to 0+} \frac{1}{x} = \infty \quad \text{and} \quad \lim_{x \to 0-} \frac{1}{x} = -\infty$$



Defn :- A line $y = b$ is said to be a horizontal asymptote of $y = f(x)$ if either

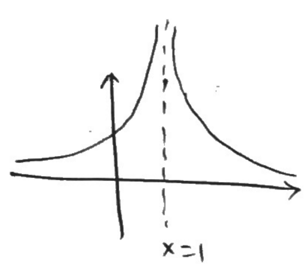
$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b.$$

– A line $x = a$ is said to be a vertical asymptote of $y = f(x)$ if either

$$\lim_{x \to a+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$

Exm : $\quad f(x) = \dfrac{1}{(x-1)^2}$

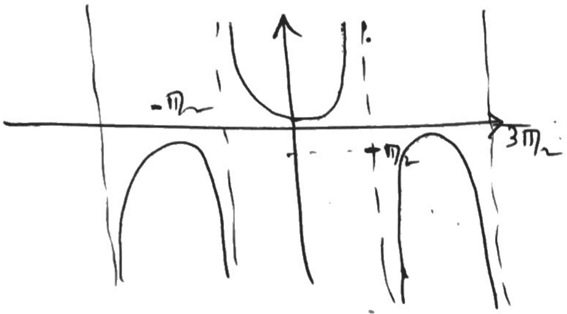
$$\lim_{x \to 1+} f(x) = \infty \qquad \lim_{x \to 1-} f(x) = \infty$$

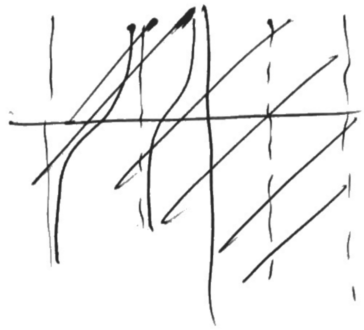


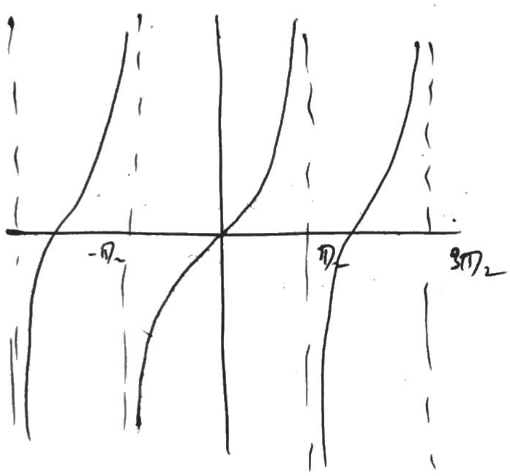
Also, $\lim_{x \to \infty} f(x) = 0$

So, $x = 1$ vertical & $y = 0$ horizontal asymptotes.

$$y = \sec x = \frac{1}{\cos x} \quad , \quad y = \tan x = \frac{\sin x}{\cos x}$$







$$\lim_{x \to \frac{(2n+1)\pi}{2}^{+}} \sec x = \pm\infty$$

$x = \frac{(2n+1)\pi}{2}$ Vertical asymptote.

$$y = \frac{x+4}{x-1}$$

$\lim_{x \to 1} y = \pm\infty$ So, $x = 1$ is the vertical asymptote

$y = 1 + \frac{5}{x-1}$ So, $\lim_{x \to \infty} y = 1$. Therefore, $y = 1$ is the horizontal asymptote.

H.W $\quad y = \frac{1}{x^2 - 9}$, $x = \pm 3$, $y = 0$

\# $\quad y = 1 + 2\frac{\sin x}{x}$

$\cancel{\lim_{x \to \infty} \left|\frac{\sin x}{x}\right| \le 1}$

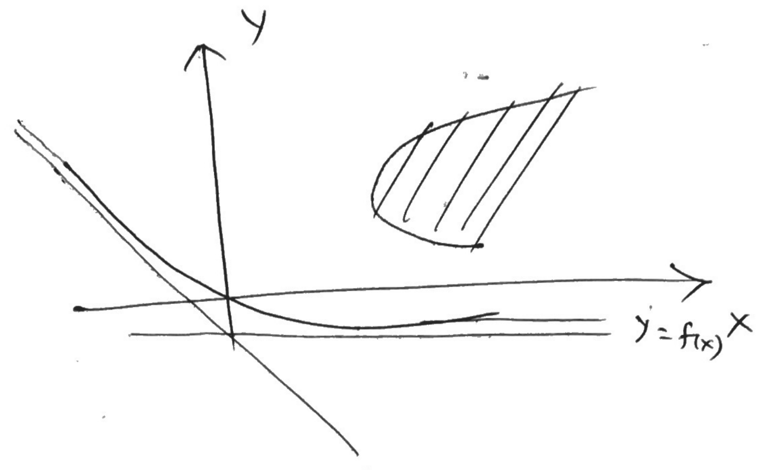
$0 \le \left|\frac{\sin x}{x}\right| \le \frac{1}{|x|}$ Hence, $\lim_{x \to \pm\infty} \frac{\sin x}{x} = 0$ (Sandwich)

So, $\lim_{x \to \pm\infty} y = 1$.

Hence, $y = 1$ is the horizontal asymptote.

Oblique Asymptote :-

If the asymptote to a curve is not parallel to x or y axis, it is called oblique asymptote.



$\Rightarrow$ $f(x) = \dfrac{P(x)}{Q(x)}$ $\qquad P(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$

$Q(x) = b_0 x^{n-1} + b_1 x^{n-2} + \dots + b_{n-1}$

The degree of $P(x) + 1$ = The degree of $Q(x)$.

Asymptote $\quad y = ax + b + \dfrac{1}{g(x)}$

$$a = \lim_{x \to \pm\infty} \frac{f(x)}{x}$$

$$b = \lim_{x \to \pm\infty} [f(x) - ax]$$

$y = \dfrac{x^2}{\sqrt{x^2 - 9}}$

$\lim_{x \to \pm 3} y = \infty$ $\quad$ So, $x = \pm 3$ are vertical asymptotes.

$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{1}{\sqrt{1 - 9/x^2}} = 1$$

$$b = \lim_{x \to \infty} (f(x) - x) = \lim_{x \to \infty} \left(\frac{x^2}{\sqrt{x^2 - 9}} - x \right) = 0$$

So $y = x$ is an oblique asymptote.

Also, $y = -x$ is an " " . (By symmetry)