[redacted]

Curve Sketching or Curve tracing :

Sketch or trace the graph !

$y = f(x)$ or $f(x,y) = 0$

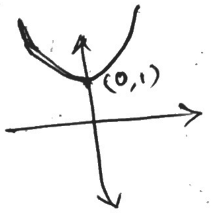
$y = x^2$, $y^2(x^2+1) = (1-x^2)$

$\downarrow$

$|x| < 1$.

1. Domain & range

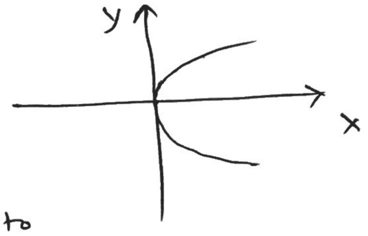
2. Intersection points with the axes. $y = x^2+1$



3. Check Symmetry

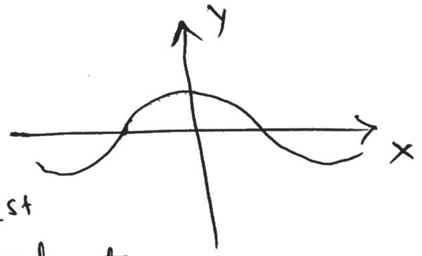
i) $g(x,-y) = g(x,y)$: symmetric w.r.to x-axis

$y^2 = x$



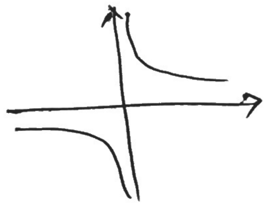
ii) $g(-x,y) = g(x,y)$: symm. w.r.to y-axis.

$y = \cos(x)$.



iii) $f(x,y) = f(-x,-y)$: symm in 1st and 3rd quadrant.

$y = \frac{1}{x}$.



4. Check whether the fn is even or odd.

$f(x) = f(-x)$ or $f(-x) = -f(x)$.

$y = \cos(x)$, x^3 $y = \sin(x)$, x^2

6. Convexity or Concavity:

determine interval in which f'' +ve or -ve.

7. Find points of inflection: $f''=0$, if exists.

5. Critical points, maxima & Minima.

8. Find asymptotes.

9. Draw the graph.

EXM:- $y = \dfrac{\ln x}{x}$

1. Domain: $(0, \infty)$, $\underline{\underline{x > 0}}$

2. The curve does not intersect y-axis $(x=0)$

$y = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1.$

So, the curve intersects x-axis $(y=0)$ at $(1,0)$

3&4 $f(x)$ is only defined for $x > 0$, so not even or odd & not symm. w.r.to ~~...~~ x-axis. $(f(x,-y) \neq f(x,y))$

5. $y' = \dfrac{1-\ln(x)}{x^2} = 0 \Rightarrow \ln x = 1$

$\Rightarrow x = e.$

$y'' = \dfrac{2\ln x - 3}{x^3}.$

$\Rightarrow y''\big|_{x=e} = -\dfrac{1}{e^3} < 0$ So, the curve has a maxima at $x=e$ and the value is $= 1/e$.

6. $y'' = \dfrac{2 \ln x - 3}{x^3} = 0$

$\Rightarrow \ln x = 3/2 \Rightarrow x = e^{3/2}$

This implies :

$y'' < 0$ in $(0, e^{3/2})$, so concave

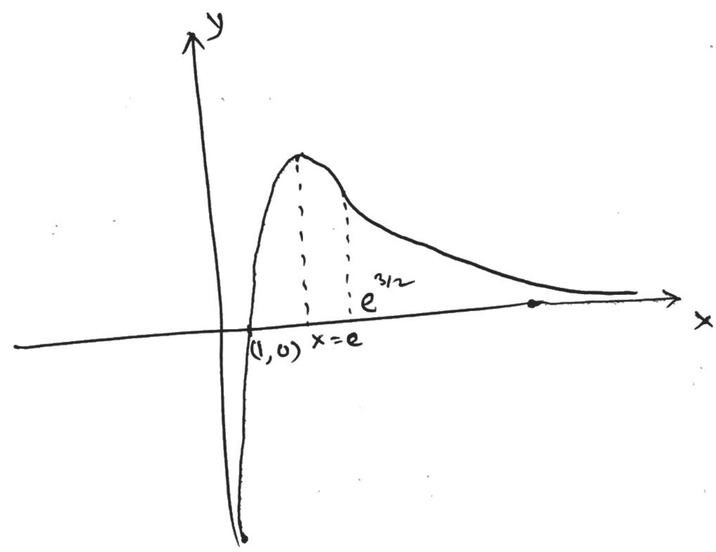
and $y'' > 0$ in $(e^{3/2}, \infty)$, so convex.

And, $x = e^{3/2}$ will give the point of inflection.

i.e. $\left(e^{3/2}, \frac{3}{2} e^{-3/2}\right)$ is a pt. of inflection.

8. $\lim\limits_{x \to \infty} \dfrac{\ln x}{x} = 0$ and $\lim\limits_{x \to 0+} \dfrac{\ln x}{x} = -\infty$

So, $y = 0$ is a horizontal asymptote & $x = 0$ is a vertical asymptote.



H.W $y = x - \frac{1}{x} = \frac{x^2-1}{x}$

ExM: $y = x^{5/3} - 3y^{2/3} = x^{2/3}(x-3)$.

1. Domain $= \mathbb{R}$

2. Intersects x-axis at $x=0$ & $x=3$

3. No symmetry. 4. Not even or odd.

5. $y' = \dfrac{5x-6}{3x^{1/3}}$

Critical pt. : $y'=0 \Rightarrow x = 6/5$,

Not derivable at $x=0$.

$y'' = \dfrac{(10x+6)}{9x^{4/3}}$

$y''\big|_{x=6/5} > 0$ So, local minima at $x = 6/5$.

In $(-\delta, \delta)$: $f(x) \leq f(0)$ $\therefore$ $x=0$ has a local maxima.

6. $y'' = 0 \Rightarrow x = -3/5$.

in $(-\infty, -3/5)$ $y'' < 0$: Concave

In $(-\frac{3}{5}, 0)$, $y'' > 0$: Convex.

In $(0, \infty)$, $y'' > 0$: Convex.

So, $x = -3/5$ gives pt. of inflection, Not $\underline{x=0}$.

$\left(-\frac{3}{5}, -\frac{18}{5}\left(\frac{3}{5}\right)^{2/3}\right)$