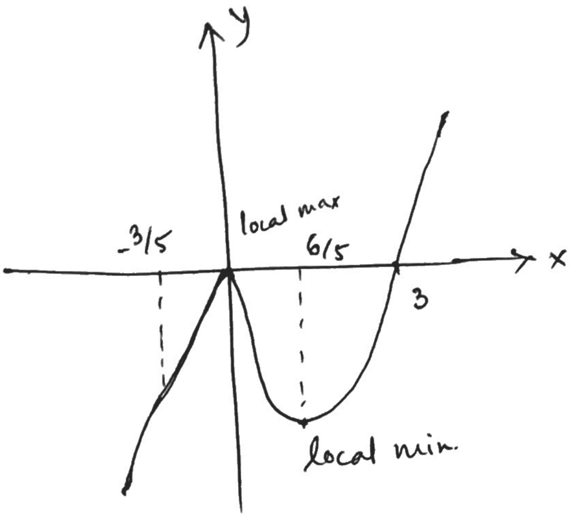
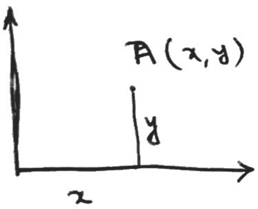
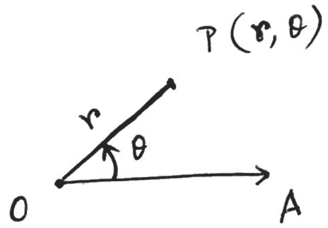
7. No asymptotes.



Polar Co-ordinates:





θ : +ve when measured in counterclockwise direction.

O : pole $(0, \theta)$ θ is not definite for pole

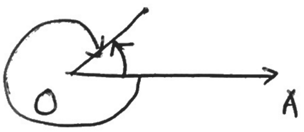
OA : polar axis.

In general, the angle is not unique

θ, $2n\pi + \theta$ are same

But, in convention, $-\pi < \theta \leq \pi$ or $0 \leq \theta < 2\pi$

$P(2, \pi/3)$ or $(2, -\frac{5\pi}{3})$ $0 \leq r < \infty$

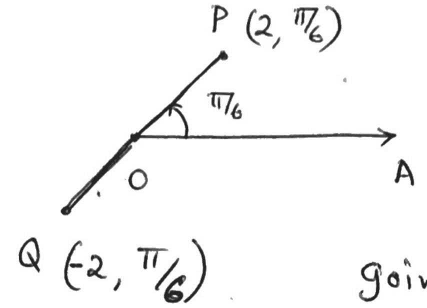


↑ To make unique

Negative r :

$\frac{3\pi}{4} + 2\pi$

$\frac{3\pi + 8\pi}{4}$



going backward

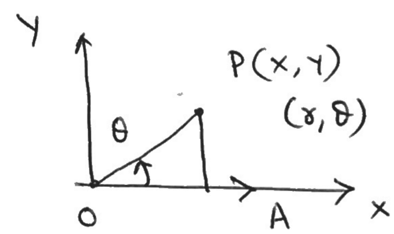
$(r, 2n\pi + \theta)$, $n = 0, \pm 1, \pm 2, \ldots$

or $(-r, -\pi + \theta - 2n\pi)$, $n = 0, \pm 1, \pm 2, \ldots$ one the same point

Exm: $(1, \pi/4) \equiv \left(-1, -\frac{3\pi}{4}\right) \equiv \left(1, \frac{9\pi}{4}\right) \equiv \left(-1, -\frac{11\pi}{4}\right)$

The representation is not Unique!

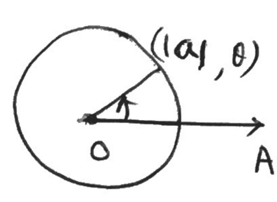
Polar & Cartesian



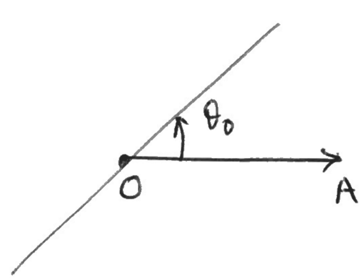
$x = r\cos\theta$, $y = r\sin\theta$

$x^2 + y^2 = r^2$, $\frac{y}{x} = \tan\theta$

Circle

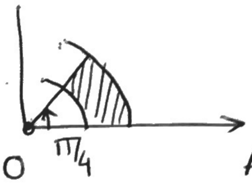


$r = a$, represents a circle of radius $|a|$.

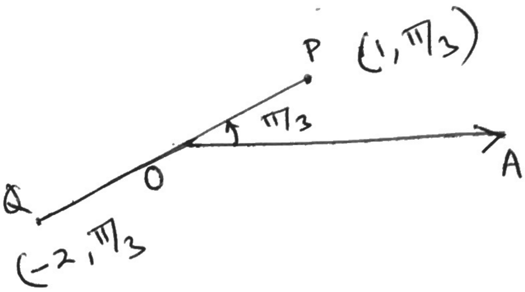


$\theta = \theta_0$ is a straight line passing through origin, makes an angle θ_0 with ~~the~~ polar axis.

(i) $1 \le r \le 2$, $0 \le \theta \le \pi/4$



(ii) $-2 \le r \le 1$, $\theta = \pi/3$



Exm :-

$r = 1 - \sin\theta$

$r^2 = r - r\sin\theta$

$\Rightarrow x^2 + y^2 + y = r$

$\Rightarrow (x^2+y^2+y)^2 = x^2+y^2$

$\Rightarrow x^4 + y^4 + \cancel{y^2} + 2x^2y^2 + 2y^3 + 2x^2y - x^2 = 0$

Exm :

$x^2 + (y-2)^2 = 4$

$\Rightarrow x^2 + y^2 - 4y + 4 = 4$

$\Rightarrow x^2 + y^2 - 4y = 0$

$\Rightarrow r^2 - 4r\cos\theta \Rightarrow r^2 = 4r\cos\theta \Rightarrow r = 4\cos\theta$

Exm :

$(5, \tan^{-1}(4/3)) \longrightarrow (x, y)$
(r, θ)

$\tan\theta = 4/3 \Rightarrow \sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

$x = r\cos\theta = 5 \cdot 3/5 = 3$

$y = r\sin\theta = 5 \cdot 4/5 = 4$.

Exm :

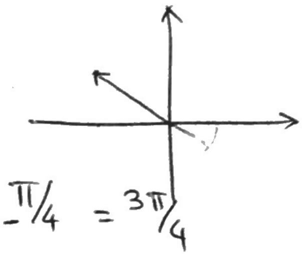
$(-3, 2\pi) \to (x, y)$

$x = -3\cos\theta = -3$

$y = -3\sin(2\pi) = 0$.

$(-3, 0)$

Exm: $(-1,1) \to (r,\theta)$



$r = \sqrt{(-1)^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(-1) = \pi - \pi/4 = 3\pi/4$

2nd co-ordinate. $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ or $\left(-\sqrt{2}, -\pi/4\right)$

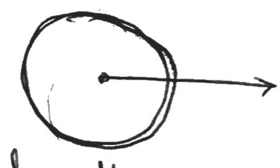
Graph

$r = f(\theta)$ or $F(r,\theta) = 0$

Symmetry:

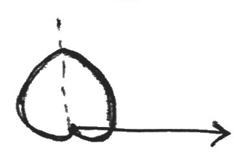
a) $\theta \to -\theta$: if the eqn remains unchanged, the curve is symm. about the polar axis.

b) $r \to -r$ or $\theta \to \theta + \pi$: if the eqn remains unchanged, the curve is symm. about the pole.



c) $\theta \to \pi - \theta$: ~~the~~ if unchanged, the curve is symm. about $\theta = \pi/2$.

$r = 1 + \sin\theta$



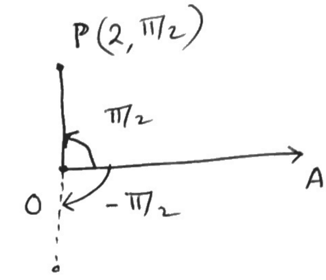
Slope

$r = f(\theta)$.

$x = r\cos\theta = f(\theta)\cos\theta \qquad y = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'\sin\theta + f\cos\theta}{f'\cos\theta - f\sin\theta}, \quad \frac{dx}{d\theta} \neq 0$$

Is $(2, \pi/2)$ on $r = 2\cos(2\theta)$?



$r = 2\cos(\pi) = -2$

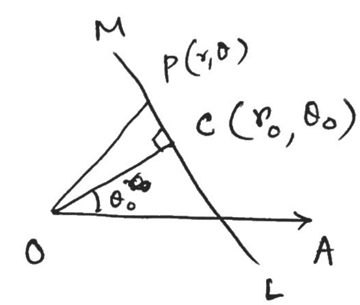
So, apparently $(2, \pi/2)$ is not on $r = 2\cos(2\theta)$

Let us check $(-2, -\pi/2)$.

$r = 2\cos(\pi) = -2$. So, $(-2, -\pi/2)$ is on $r = 2\cos(2\theta)$

But, $(-2, -\pi/2) \equiv (2, \pi/2)$, the same point.

Straight Line :-



$$\frac{r_0}{r} = \cos(\theta - \theta_0)$$

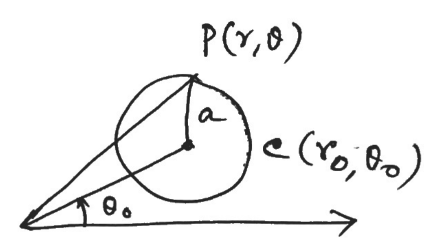
$$\Rightarrow \underline{r\cos(\theta - \theta_0) = r_0}$$

Line passing through origin, $r_0 = 0$

$$\Rightarrow r\cos(\theta - \theta_0) = 0$$

$$\Rightarrow \underline{\theta = \theta_0 + \pi/2}$$

Circle



$$a^2 = r^2 + r_0^2 - 2rr_0\cos(\theta - \theta_0)$$

If the circle passes through the origin, $r_0 = a$

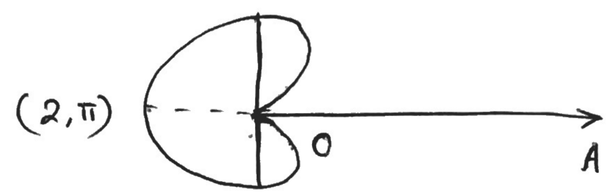
$$\Rightarrow a^2 = a^2 + r^2 - 2ra\cos(\theta - \theta_0)$$

$$\Rightarrow \underline{r = 2a\cos(\theta - \theta_0)}.$$

Cardioid

$$r = 1 - \cos\theta$$

$\theta \to -\theta$: ✓ symm. w.r.to polar axis.



$$r = 1 + \cos\theta$$