A. Partial Conversion

$$r = f(\theta)$$

$$\left.\begin{aligned} x &= r\cos\theta = f(\theta)\cos\theta \\ y &= r\sin\theta = f(\theta)\sin\theta \end{aligned}\right\}$$ and then find the nature of x, y.

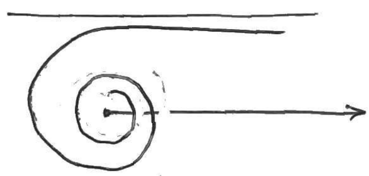
Exm. $r = \dfrac{a}{\theta}$,

$$x = r\cos\theta = \frac{a\cos\theta}{\theta} \to \infty \quad \text{as } \theta \to 0+$$

$$y = r\sin\theta = \frac{a\sin\theta}{\theta} \to a \quad \text{as } \theta \to 0+$$

i.e. $\lim\limits_{x\to\infty} y = a \Rightarrow y = a$ is an asymptote

$$\boxed{r\sin\theta = a}$$



Exm $r = a\tan\theta$,

$$x = r\cos\theta = a\sin\theta \to \pm a \quad \text{as } \theta \to \pm\frac{\pi}{2}$$

$$y = r\sin\theta = a\frac{\sin^2\theta}{\cos\theta} \to \infty \quad \text{as } \theta \to \pm\frac{\pi}{2}$$

i.e. $\lim\limits_{x\to a} y = \infty$, $\lim\limits_{x\to -a} y = \infty$

$$\boxed{r\cos\theta = \pm a}$$

$\Rightarrow$ $x = a$ is an asymptote
& $x = -a$ is another "

Exm. $r = a(\sec\theta + \tan\theta)$

$$x = a(1+\sin\theta) \to 2a \quad \text{as } \theta \to \frac{\pi}{2}$$

$$y = a\frac{(1+\sin\theta)\sin\theta}{\cos\theta} \to \infty \quad \text{as } \theta \to \frac{\pi}{2}$$

i.e. $\lim\limits_{x\to 2a} y = \infty$

So, $x = 2a$ is a vertical asymp.

B. Full Conversion

$$r = f(\theta) \to y = f(x).$$

Exm. $r\sin\theta = 2\cos(2\theta)$

$$\Rightarrow y = 2(\cos^2\theta - \sin^2\theta) \Rightarrow (x^2+y^2)y = 2(x^2-y^2)$$

$$\Rightarrow x^2(y-2) + y^3 + 2y^2 = 0$$

Asymptotes: $y = 2$ horizontal.

No vertical Asymp.

No oblique asymptote as

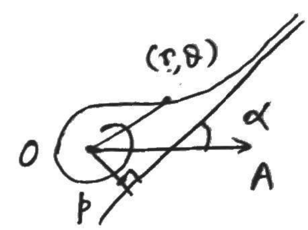
$$x = r\cos\theta = \frac{2\cos(2\theta)\cos\theta}{\sin\theta}$$

$$y = r\sin\theta = 2\cos(2\theta)$$

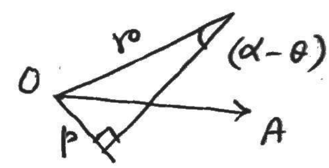
both x, y does not go to infinity simultaneously.

C. <u>Polar Formula :-</u>

1. $$\alpha = \lim_{r \to \infty} \theta$$



2. $$p = \lim_{\theta \to \alpha} \frac{r^2}{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} = r\sin(\alpha - \theta).$$



<u>ExM.</u>

$$r\sin(n\theta) = a$$

$$\alpha = \lim_{r \to \infty} \theta = \lim_{r \to \infty} \frac{1}{n}\sin^{-1}\left(\frac{a}{r}\right) = \frac{m\pi}{n}, \quad m \in \mathbb{Z}.$$

$$\frac{dr}{d\theta}\sin(n\theta) + r\cos(n\theta).n = 0$$

$$\therefore \frac{dr}{d\theta} = -\frac{rn\cos(n\theta)}{\sin(n\theta)}$$

$$\therefore \lim_{\theta \to \frac{m\pi}{n}} \frac{r^2}{\sqrt{r^2 + \frac{r^2 n^2\cos^2(n\theta)}{\sin^2(n\theta)}}} = \lim_{n\theta \to m\pi} \frac{a/\sin(n\theta)}{\frac{\sqrt{\sin^2(n\theta) + n^2\cos^2(n\theta)}}{\sin(n\theta)}}$$

$$= \frac{a}{n}.$$

So, the asymptotes are:

$$\underline{r\sin\left(\frac{m\pi}{n} - \theta\right) = \frac{a}{n}}$$