Mathematics I–Section 3 Tutorial 1 July 31st 2017

Name: Key **Roll No:**

Try each problem in your own. You should maintain a notebook for Tutorial problems.

1. Compute the following limits:
   i) $\lim_{x\to0}\left(\frac{1}{x\sin x}-\frac{1}{x^2}\right)$, ii) $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$, iii) $\lim_{x\to\infty}\frac{e^x}{x^n}$.

i) $$\lim_{x\to0}\left(\frac{1}{x\sin x}-\frac{1}{x^2}\right) = \lim_{x\to0}\frac{x-\sin x}{x^2\sin x} \underset{\left(\frac{0}{0}\right)}{\overset{L'H}{=}} \lim_{x\to0}\frac{1-\cos x}{x^2\cos x+2x\sin x} \underset{\left(\frac{0}{0}\right)}{\overset{L'H}{=}} \lim_{x\to0}\frac{\sin x}{2\sin x+2x\cos x-x^2\sin x+2x\cos x}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L'H}{=}} \lim_{x\to0}\frac{\cos x}{2\cos x+4\cos x-4x\sin x-2x\sin x-x^2\cos x} = \frac{1}{6}\text{ (Ans)}$$

ii) $$y=\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} \Rightarrow \ln y = \lim_{x\to\infty} bx\ln\left(1+\frac{a}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\ln(1+a/x)}{1/bx}\quad\left(\frac{0}{0}\right)$$

$$\overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{1+a/x}\cdot\left(-a/x^2\right)}{\left(-1/bx^2\right)} = \lim_{x\to\infty}\frac{ab}{1+\frac{a}{x}} = ab$$

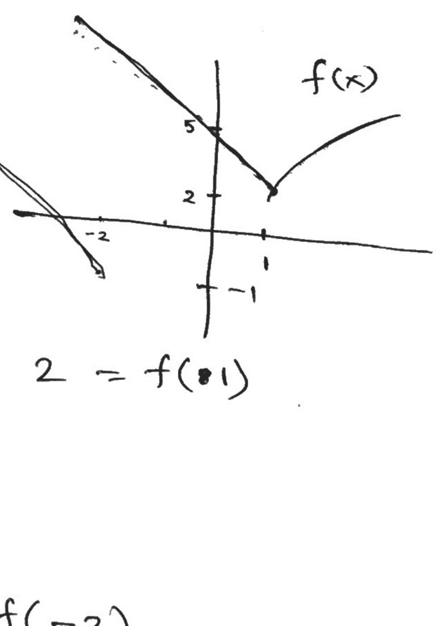
$$\Rightarrow \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

iii) $$\lim_{x\to\infty}\frac{e^x}{x^n} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L'H}{=}} \lim_{x\to\infty}\frac{e^x}{nx^{n-1}} \underset{(n-1)\text{ times}}{\overset{L'H}{=}} \lim_{x\to\infty}\frac{e^x}{n!} = \infty$$

exponential fn grows faster than polynomials.

2. Check continuity of the following function:

$$f(x)=\begin{cases}\dfrac{x-1}{\sqrt{x}-1}, & \text{if } x>1,\\ 5-3x, & \text{if } -2\le x\le 1,\\ \dfrac{6}{x-4}, & \text{if } x<-2.\end{cases}$$



Is f differentiable at $x=1$ and $x=-2$.

$$\lim_{x\to1+} f(x) = \lim_{x\to1+}\frac{x-1}{\sqrt{x}-1} = \lim_{x\to1+}\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{(\sqrt{x}-1)} = 2 = f(1)$$

$$\lim_{x\to1-} f(x) = \lim_{x\to1-}(5-3x) = 2 = f(1)$$

f is continuous at $x=1$

$$\lim_{x\to-2+} f(x) = \lim_{x\to-2+}(5-3x) = 5+6 = 11 = f(-2)$$

$$\lim_{x\to-2-} f(x) = \lim_{x\to-2-}\frac{6}{x-4} = \frac{6}{-6} = -1 \ne f(-2)$$

f is not continuous at $x=-2$. (So f is not diff. at $x=-2$)

Check for $x=1$: $$\lim_{h\to0+}\frac{f(1+h)-f(1)}{h} = \frac{1}{2} \ne \lim_{h\to0-}\frac{f(1+h)-f(1)}{h} = -3$$