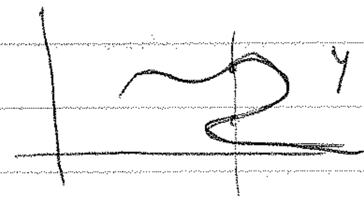
10.1 Parametric Curves

- Difficult to express as a function.

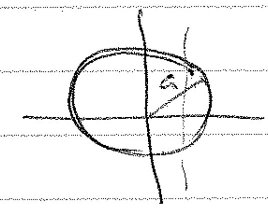


$y = f(x)$

a circle:

$$x^2 + y^2 = a^2$$

$$\Rightarrow \quad y = \pm\sqrt{a^2 - x^2}$$



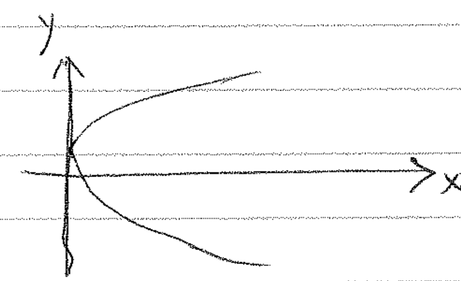
$x = f(t)$

$y = g(t)$ functions of time. t. (Parameter)

$$(x, y) = (f(t), g(t)) \quad , \quad a \le t \le b$$

Exm:- $x = t^2$, $y = t + 1$, $-\infty < t < \infty$.

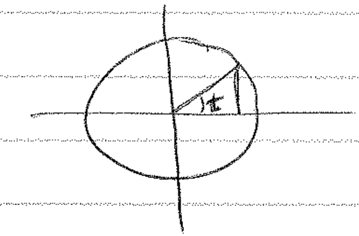
t	x	y
-2	4	-1
-1	1	0
0	0	1
$+1$	1	2
2	4	3



$\underline{x = (y-1)^2}$ $\underline{\text{Parabola}}$

\# $x = \cos t, \quad y = \sin t, \quad 0 \le t \le 2\pi$

$x^2 + y^2 = 1$, a circle of radius $\underline{1}$.

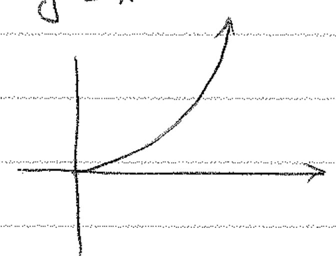


\# $x = h + r\cos t, \quad y = k + r\sin t, \quad 0 \le t \le 2\pi$

$$\underline{(x-h)^2 + (y-k)^2 = r^2}$$

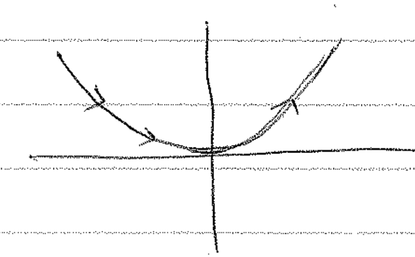
\# $x = \sqrt{t}, \quad y = t, \quad t \ge 0$

$y = x^2$



$x = t, \quad y = t^2, \quad -\infty < t < \infty$

$y = x^2$



<u>Creating Parametrization:-</u>

$$x = y^4 - 3y^2$$

Let, $y = t$, $\quad \underline{x = t^4 - 3t^2}$

\# Find the parametrization of the line ~~seg~~ segment joining (2, 1) to (3, −6) on $t \in [0, 1]$

~~So,~~ $(x(t), y(t))$, $t \in [0, 1]$

$x(0) = 2, \quad x(1) = 3$

$y(0) = 1, \quad y(1) = -6.$

$$\frac{x - x_1}{t - t_1} = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow \frac{x - 2}{t - 0} = \frac{3 - 2}{1 - 0}$$

$$\Rightarrow (x - 2) = t.$$

$$\underline{\underline{x = t + 2}}$$

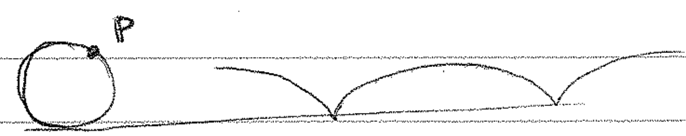
$$\frac{y - y_1}{t - t_1} = \frac{y_2 - y_1}{t_2 - t_1} \Rightarrow \frac{y - 1}{t - 0} = \frac{-6 - 1}{1 - 0}$$

$$\Rightarrow y - 1 = -7t.$$

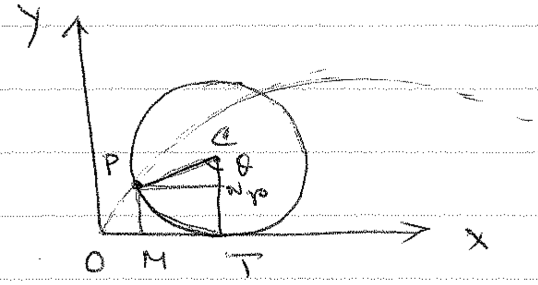
$$y = 1 - 7t$$

So, $(t + 2, 1 - 7t)$ for $\underline{0 \le t \le 1}$

\# Cycloid :- Tracing the path of a point on a circle along a st. line.



P was at the origin O for $\theta = 0$.



$OT = \widehat{PT} = r\theta$. θ = radian

$$x = OT - MT$$
$$= r\theta - r\sin\theta = r(\theta - \sin\theta)$$

$$y = CT - CN = r - r\cos\theta = r(1-\cos\theta)$$

So, the parametric equations of cycloid are:

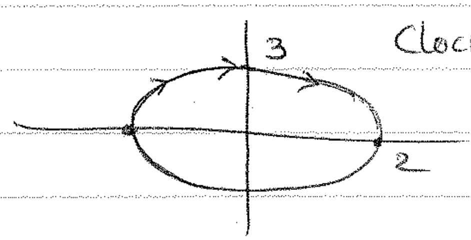
$$x = r(\theta - \sin\theta), \quad y = r(1-\cos\theta) \qquad \theta \in \mathbb{R}.$$

\# $x = -2\cos(3t)$, $y = 3\sin(3t)$ $t \in [0, \pi/3]$

$$\left(\frac{x}{-2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$\cancel{3t = \pi/2}$
$\cancel{t = \pi/6}$

$\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ Ellipse.

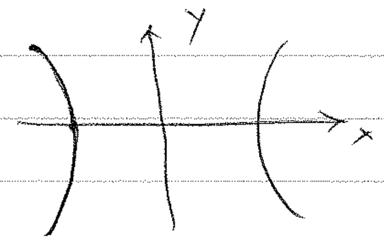


Clockwise Orientation.

& $0 \le y \le 3$

$x = -2\sec(3t)$, $y = 2\tan(3t)$ $t \in (-\pi/6, \pi/6)$

$$\left(\frac{x}{-2}\right)^2 - \left(\frac{y}{2}\right)^2 = 1$$

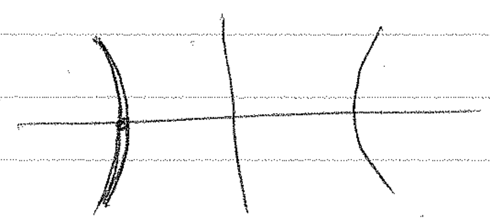


$$\Rightarrow \underline{x^2 - y^2 = 4}$$ Hyperbola

$$\underline{-\infty < y < \infty}$$

$x = -4\cosh(3t)$, $y = 4\sinh(3t)$ $\underline{-\infty < t < \infty}$

$$\left(\frac{x}{-4}\right)^2 - \left(\frac{y}{4}\right)^2 = 1$$



$$\Rightarrow \underline{x^2 - y^2 = 4^2}$$

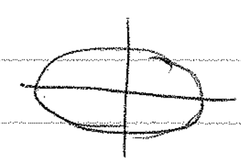
$y = x^3 + 1$ from $(-4, -63)$ to $(5, 126)$

Let, $x = t$, $y = t^3 + 1$

with $-4 \le t \le 5$

x or y is an affine function of t if $x = at + b$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$ $(5,0)$ to $(-5,0)$



Buttom part: $x = 5\cos t$

$y = -3\sin t$, $t \in [0, \pi]$.