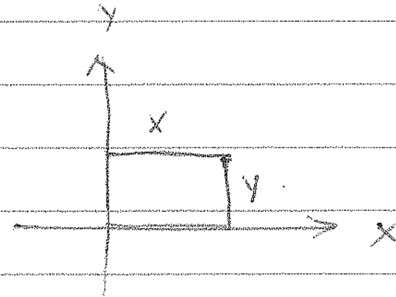
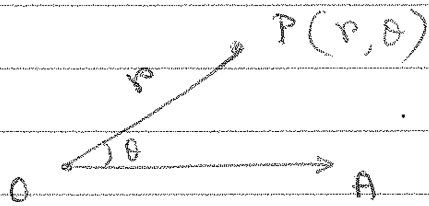
10.3

Polar Co-ordinates

Cartesian co-ordinates:



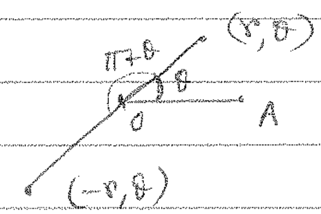
(Newton) Polar co-ordinate



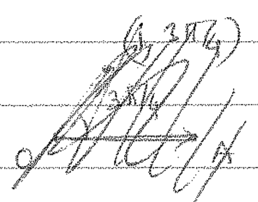
O: pole

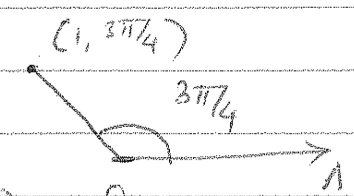
OA: polar axis.

$P \equiv O$ $(0, \theta)$ for any θ.

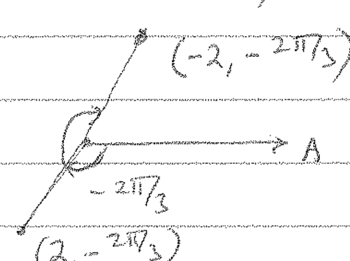


Exm: $(1, 3\pi/4)$

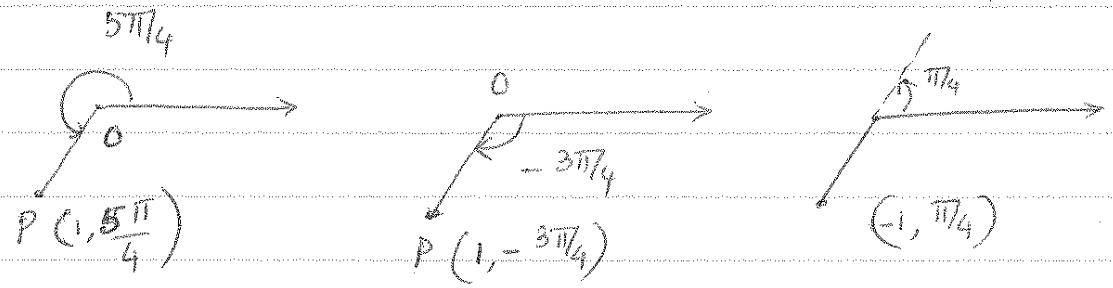


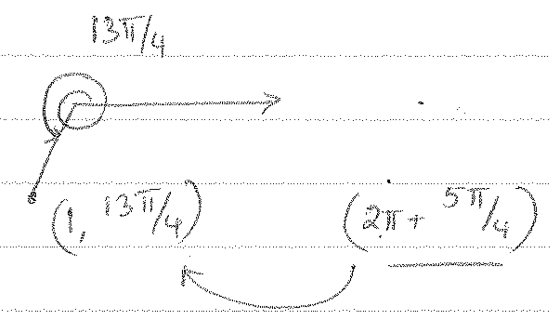


$(2, -2\pi/3)$



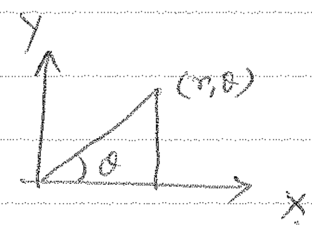
Representation is not Unique!





$$(r,\theta) \equiv (r, \theta + 2n\pi) \equiv (-r, \theta + (2n+1)\pi), \quad n = \text{integer}.$$

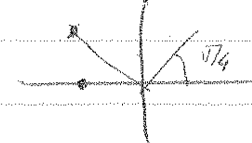
Polar in Cartesian :-



$$x = r\cos\theta$$
$$y = r\sin\theta \qquad 0 \le \theta \le \pi/2$$

$$r^2 = x^2 + y^2, \qquad \tan\theta = y/x.$$

Exm $(-1,1) \to (r,\theta)$



$r^2 = (-1)^2 + 1^2 \Rightarrow 2. \Rightarrow r = \sqrt{2}$. Choosing +ve.

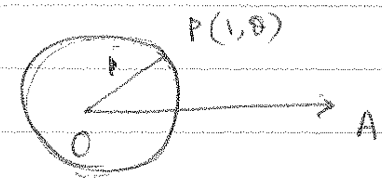
$(-1,1)$ is in 2nd coordinate. $\tan\theta = \frac{1}{-1} = -1 \Rightarrow \theta = \pi - \pi/4$
$= 3\pi/4$

$(-1,1) \to (\sqrt{2}, 3\pi/4)$. or $(-\sqrt{2}, -\pi/4)$.

Graph $\underline{r = f(\theta)}$ or $F(r,\theta) = 0$

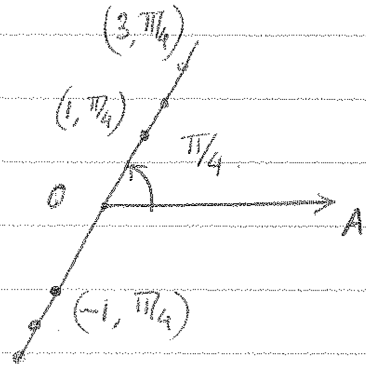
\# $r = 1$:

(θ is not restricted)



$\underline{x^2 + y^2 = 1}$: Circle

\# $\theta = \pi/4$ (r is not restricted)



$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \pi/4$

$\Rightarrow y = \tan\frac{\pi}{4} \cdot x$

$\Rightarrow \underline{y = x}$: Straight line.

\# $r = 2\sin\theta$ Change it to Cartesian co-or

$r = 2\, y/r$ $\qquad x = r\cos\theta$

$\qquad y = r\sin\theta$

$\Rightarrow r^2 = 2y$

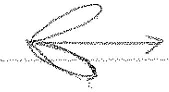
$\Rightarrow y^2 - 2y + x^2 = 0 \Rightarrow \underline{(y-1)^2 + x^2 = 1}$ Circle.

Or plot for diff. θ. $(0, \pi/4, \pi/2, \pi/3, \ldots)$

Symmetry :-

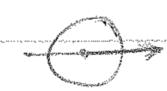
a) $\theta \to -\theta$: if the equation is ~~symmetric~~ unchanged, the

$r=1$ curve is symmetric about the polar axis.



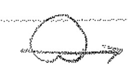
b) $r \to -r$ or $\theta \to \theta+\pi$, ~~the~~ if the equation is

$\theta=1$ unchanged, the curve is symm. about the pole.



c) $\theta \to \pi-\theta$, If the equation is unchanged, the curve is symm. about the line $\theta = \pi/2$.

$\underline{r = 1+\sin\theta}$



Tangent :- To find a tangent line to the curve : $r = f(\theta)$.

Parametric eqn : $x = r\cos\theta \Rightarrow x = f(\theta)\cos\theta$

$y = r\sin\theta \Rightarrow y = f(\theta)\sin\theta$.

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$= \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

At pole : $r=0$.

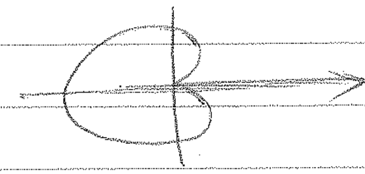
$$\Rightarrow \frac{dy}{dx} = \tan\theta \quad \text{If } \frac{dr}{d\theta} \neq 0.$$

Horizontal : $dy/d\theta = 0$.

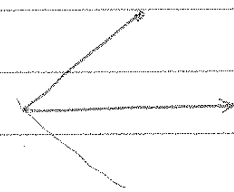
Vertical : $dx/d\theta = 0$

Cardioid :- $r = 1 - \cos\theta$.

$\theta \to -\theta$ $\quad r = 1 - \cos\theta$

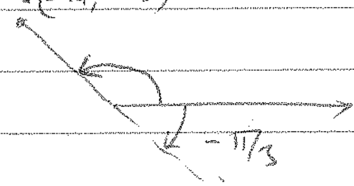


Exm :- $(7, \pi/3) \neq (-7, -\pi/3)$



$\left(14, \frac{26\pi}{3}\right) = (-14, -\pi/3)$

$\frac{26\pi}{3} = 8\pi + \frac{2\pi}{3}$



Ex # Polar $\to$ Cartesian $(4, -5\pi/2)$

$x = 4\cos(-5\pi/2) = 4\cos(2\pi + \pi/2) = 4\cos\pi/2 = 0$

$y = 4\sin(-5\pi/2) = -4\sin(2\pi + \pi/2) = -4\sin\pi/2 = -4$

$(0, -4)$

$r^2 \sin 2\theta = 9 \Rightarrow 2(r\sin\theta)(r\cos\theta) = 9$

$\Rightarrow xy = 9/2$

Hyperbola .

$r = 2\cot\theta \operatorname{cosec}\theta$

$\Rightarrow r = 2\frac{\cos\theta}{\sin\theta \sin\theta}$

$\Rightarrow r^2 \sin^2\theta = 2r\cos\theta$

$y^2 = 2x$: Parabola