Name: Key **Section:** _____

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (2 points) Write the power series in Sigma-notation: $\left(\sum_{n=0}^{\infty} a_n(x-c)^n\right)$

$$5 - \frac{5}{2}x + \frac{5}{4}x^2 - \frac{5}{8}x^3 + \frac{5}{16}x^4 - \cdots$$

$c = 0, \quad a_n = (-1)^n \cdot 5 \cdot \frac{1}{2^n}$

$$\sum_{n=0}^{\infty} (-1)^n \frac{5}{2^n} x^n$$

2. (4 points) Write the first three terms in the power series representation of $\frac{x^2}{x+3}$.

$$\frac{x^2}{x+3} = \frac{x^2}{3}\left(1+\frac{x}{3}\right)^{-1} = \frac{x^2}{3}\sum_{n=0}^{\infty}\left(-\frac{x}{3}\right)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{3^{n+1}}$$

$$\frac{x^2}{x+3} = \frac{x^2}{3} - \frac{x^3}{3^2} + \frac{x^4}{3^3} - \cdots$$

3. (4 points) Find the first two non-zero terms of the Taylor series of $f(x) = \sin(x)$ centered at $c = \pi$?

$f(x) = \sin(x) \Rightarrow f(\pi) = 0$

$f'(x) = \cos x \Rightarrow f'(\pi) = -1$

$f''(x) = -\sin x \Rightarrow f''(\pi) = 0$

$f^{(3)}(x) = -\cos x \Rightarrow f^{(3)}(\pi) = 1$

$$\sin(x) = f(\pi) + \frac{f'(\pi)}{1!}(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2 + \frac{f^{(3)}(\pi)}{3!}(x-\pi)^3 + \cdots$$

$$= -(x-\pi) + \frac{1}{3!}(x-\pi)^3 - \cdots$$