Name: KEY **Section:** _____

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (a) (2 points) Prove that $\frac{d}{dx}(e^x) = e^x$.

By Cancellation Equation, $\ln(e^x) = x$.

Differentiating, $\frac{1}{e^x}\frac{d}{dx}(e^x) = 1$

$\Rightarrow \frac{d}{dx}(e^x) = e^x$.

(or Full 6 marks if Inverse fn theorem is used)

(b) (4 points) i) Differentiate $y = x^x$. ii) Find $\int_e^2 \frac{dx}{x \ln x}$.

i) $y = x^x$.

Take ln on both sides,

$\ln y = x \ln x$.

Differentiating.

$\frac{1}{y}y' = \ln x + x \cdot \frac{1}{x}$

$\Rightarrow y' = y(1 + \ln x)$

$\therefore y' = x^x(1 + \ln x)$

(-1 if y is not replaced by x^x)

ii) $\int_e^2 \frac{dx}{x \ln x}$

$= \int_1^{\ln 2} \frac{du}{u}$

$= \ln|u| \Big]_1^{\ln 2}$

$= \ln(\ln 2)$

Let, $\ln x = u$

$\frac{1}{x}dx = du$.

(+1 if the substitution is correct)

or $\ln(|\ln 2|)$.

2. (4 points) Solve $\frac{dx}{dt} = \frac{t}{x}$, $x(0) = -3$.

$\frac{dx}{dt} = t/x$.

$\Rightarrow x\,dx = t\,dt$.

$\Rightarrow \frac{x^2}{2} = \frac{t^2}{2} + c$.

$\Rightarrow x^2 = t^2 + 2c$.

$x(0) = -3 \Rightarrow (-3)^2 = 0^2 + 2c$

$\Rightarrow c = 9/2$.

So, $x^2 = t^2 + \frac{9}{2} \cdot 2 \Rightarrow x^2 = t^2 + 9 \Rightarrow x = \pm\sqrt{t^2+9}$

-1 if until $x^2 = t^2 + 9$.

-1 if ~~until~~ $x = \sqrt{t^2+9}$

-2 if until $x^2 = t^2 + 2c$