Name: Key. Section: ____________

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (a) (2 points) Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$.

Let, $u = \cos x$
$du = -\sin x\, dx$

$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$$
$$= -\int_1^0 \frac{du}{1+u^2}$$
$$= \int_0^1 \frac{du}{1+u^2} = \tan^{-1} u\Big]_0^1 = \frac{\pi}{4}.$$

(b) (4 points) Differentiate i) $y = \sinh(\cos x)$. ii) $y = \arccos(\exp(x/10))$.

i) $y = \sinh(\cos x)$

$$\frac{dy}{dx} = \cosh(\cos x)\cdot(-\sin x)$$
$$\therefore\ y' = -\cosh(\cos x)\cdot \sin x$$

ii) $y = \cos^{-1}\left(e^{x/10}\right)$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-(e^{x/10})^2}}\cdot \frac{1}{10}e^{x/10}$$
$$\therefore\ y' = -\frac{1}{\sqrt{1-e^{x/5}}}\cdot\frac{e^{x/10}}{10}.$$

2. (a) (1 point) Convert $\lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x^2}\right)$ into $\frac{0}{0}$ form.

$$\lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x^2}\right) \quad (\infty - \infty)$$
$$= \lim_{x\to 0}\left(\frac{x-\sin x}{x^2 \sin x}\right) \quad \left(\frac{0}{0}\right)$$

Give +3 if part a is incorrect, but (b) is correct!

(b) (3 points) Compute the limit by l'Hospital's Rule.

$$\lim_{x\to 0} \frac{x - \sin x}{x^2 \sin x}$$
l'H
$$= \lim_{x\to 0} \frac{1-\cos x}{2x\sin x + x^2\cos x} \quad \left(\frac{0}{0}\right)$$
l'H
$$= \lim_{x\to 0} \frac{+\sin x}{2\sin x + 2x\cos x + 2x\cos x - x^2\sin x} \quad \left(\frac{0}{0}\right)$$
l'H
$$= \lim_{x\to 0} \frac{\cos x}{2\cos x + 4\cos x - 4x\sin x - 2x\sin x - x^2\cos x} = \frac{1}{6}.$$