Name: Key **Section:** _____

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (5 points) Integrate by parts: $\int e^x \cos x\, dx$.

$I = \int e^x \cos x\, dx$ $\quad u(x) = \cos x$, $v(x) = e^x$.

$= \cos x \int e^x dx + \int \left(\sin x \int e^x dx\right) dx$

$= e^x \cos x + \int e^x \sin x\, dx$.

$= e^x \cos x + \sin x \int e^x dx - \int \left(\cos x \int e^x dx\right) dx$.

$= e^x(\cos x + \sin x) - \int e^x \cos x\, dx + 2c$ (constant)

$\Rightarrow I = e^x(\cos x + \sin x) - I + 2c$.

$\therefore I = \frac{1}{2} e^x(\cos x + \sin x) + c$.

[-0.5 if final integration constant 'c' is not put]

2. (a) (1 point) What will be a good substitution to integrate: $\int \frac{1}{\sqrt{x^2-4}}\, dx$?

~~$x = 2\sin\theta$~~ $x = 2\sec\theta$ or $x = 2\cosh\theta$.

(b) (4 points) Evaluate the above integration.

$\int \frac{dx}{\sqrt{x^2-4}}$ Let, $x = 2\sec\theta$, $dx = 2\sec\theta\tan\theta\, d\theta$

$= \int \frac{2\sec\theta\tan\theta\, d\theta}{2\tan\theta}$

$= \ln|\sec\theta + \tan\theta| + c$

$= \ln\left|\frac{x}{2} + \tan\left(\sec^{-1}\left(\frac{x}{2}\right)\right)\right| + c$.

or $\ln\left|\frac{x + \sqrt{x^2-4}}{2}\right| + c$.

Or

$\int \frac{dx}{\sqrt{x^2-4}}$ Let, $x = 2\cosh\theta$, $dx = 2\sinh\theta$

$= \int \frac{2\sinh\theta}{2\sinh\theta}\, d\theta$

$= \theta + c$

$= \cosh^{-1}\left(\frac{x}{2}\right) + c$.