

Name: _____

Section: _____

Clear your desk of everything except pens, pencils and erasers. Show all your work.

If you have a question raise your hand and I will come to you.

1. (5 points) Integrate: $\int \frac{dx}{1+e^x}$.

$$\begin{aligned} \int \frac{dx}{1+e^x} &= \int \frac{e^{-x} dx}{e^{-x} + 1} \\ &= - \int \frac{du}{1+u} \\ &= - \ln|1+u| + C \\ &= - \ln|1+e^{-x}| + C \\ &= - \ln\left|1+\frac{1}{e^x}\right| + C \\ &= \underline{x - \ln(1+e^x) + C} \end{aligned}$$

$$\begin{aligned} \text{Let, } e^{-x} &= u \\ -e^{-x} dx &= du \end{aligned}$$

2. (5 points) Apply comparison test to show that $\int_1^\infty e^{-x^2} dx$ converges.

$$e^{-x^2} \leq e^{-x} \quad \text{on } [1, \infty)$$

$$\text{And } \int_1^\infty e^{-x} dx = -[e^{-x}]_1^\infty = -(0 - e^{-1}) = e^{-1}, \text{ converges}$$

So, by comparison test, $\int_1^\infty e^{-x^2} dx$ converges

[If $\int_1^\infty e^{-x} dx$ convergent not shown, -2 will be deducted]