Name: Key **Section:**

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (4 points) Find the limit of the sequence $\{\sqrt[n]{5n}\}_n$.

To find $\lim_{n\to\infty} (5n)^{1/n}$.

Consider $y = \lim_{x\to\infty} f(x)$

with $f(x) = (5x)^{1/x}$.

$\ln y = \lim_{x\to\infty} \ln(5x)^{1/x}$

$= \lim_{x\to\infty} \frac{1}{x}\ln(5x)$ $\left(\frac{\infty}{\infty}\right)$

L'Hospital $= \lim_{x\to\infty} \frac{\frac{\cancel{5}}{\cancel{5}x}}{1} = 0$.

$\therefore y = e^0 = 1$ So, $\lim_{n\to\infty} (5n)^{1/n} = 1$

[-1 if l'Hospital is used on $\lim_{n\to\infty} \frac{\ln(5n)}{n}$ in discrete case.] without considering $f(x)$.

2. (6 points) Test whether the series are convergent/divergent. (Mention the test as well):
i) $\sum \frac{5-2n}{3+5n}$, ii) $\sum_{n=2}^{\infty} \frac{1}{n\ln^2 n}$, iii) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$.

[Give full marks, even if the test name and convergent/divergent ~~result~~ results are OK]

i) $\sum \frac{5-2n}{3+5n}$, $a_n = \frac{5-2n}{3+5n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5-2n}{3+5n} = -\frac{2}{5} \neq 0$.

By nth term test (or Test of divergence), $\sum a_n$ diverges.

ii) $\sum_{n=2}^{\infty} \frac{1}{n\ln^2 n}$ Consider $\int_2^\infty \frac{dx}{x(\ln x)^2}$ Let, $\ln x = u$, $\frac{1}{x}dx = du$

$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{1}{u^2}\,du$

$= -\lim_{t\to\infty} \left[\frac{1}{u}\right]_{\ln 2}^{\ln t} = -\lim_{t\to\infty}\left[\frac{1}{\ln t} - \frac{1}{\ln 2}\right] = \frac{1}{\ln 2} < \infty$

So, by integral test, the series converges.

iii) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ This is a p-series with $p = 2/3 < 1$. $\left(\sum \frac{1}{n^p}\right)$

So, by p-series test (or by integral test), the series in divergent.