Name: Key Section: ____________

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (5 points) Use limit comparison test to prove the convergence of $\sum_{n=1}^{\infty} \frac{5n^2+7n+5}{9n^{10}+2n+9}$.

Consider $\sum_{n=1}^{\infty} b_n$ with $b_n = \frac{1}{n^8}$, $n \geq 1$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5n^2+7n+5}{9n^{10}+2n+9} \cdot \frac{n^8}{1} = \lim_{n\to\infty} \frac{5n^{10}+7n^9+5n^8}{9n^{10}+2n+9}$$

$$= 5/9 < \infty.$$

Being a p-series with $p=8>1$, $\sum b_n$ is convergent.
So, by limit comparison test, $\sum a_n$ or the original series converges.

[-1 if $\sum b_n$ convergence is not mentioned]

2. (5 points) Prove that $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is absolutely convergent (AC). (Hint: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$)

$\sum_{n=1}^{\infty} a_n$ with $a_n = \frac{n!}{n^n}$.

[Full marks even if one considers $\frac{a_{n+1}}{a_n}$]

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!}\right|$$

$$= \lim_{n\to\infty} \frac{(n+1)\, n^n}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n} = \frac{1}{e} < 1.$$

So, by ratio test, $\sum a_n$ is absolutely convergent.