Name: Key Section: ____________

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (3 points) Find the sum of the series: $\sum_{n=0}^{\infty} \frac{3-2^{n+1}}{7^n}$.

$$\sum_{n=0}^{\infty} \frac{3-2^{n+1}}{7^n} = \sum_{n=0}^{\infty} \frac{3}{7^n} - \sum_{n=0}^{\infty} 2\left(\frac{2}{7}\right)^n$$

$$= \frac{3}{1-\frac{1}{7}} - \frac{2}{1-\frac{2}{7}}$$

$$= \underline{\underline{\frac{21}{6} - \frac{14}{5}}} \quad \text{or} \quad = \underline{\underline{\frac{21}{30}}} = \underline{\underline{\frac{7}{10}}}$$

[Any of the ~~two~~ 3 underlined acceptable

2. (3 points) Is the series $\sum_{n=0}^{\infty} \sin(1/n)$ convergent? Just mention the test for your result.

i) No, $\sum \sin\left(\frac{1}{n}\right)$ is divergent.

ii) By comparison test.

Additional:- $\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = \underline{1}$.

So, $\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1$. As, $\sum \frac{1}{n}$ is divergent, $\sum \sin\left(\frac{1}{n}\right)$ is div.

3. (4 points) What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$?

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} u_n$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)!} \frac{n!}{x^n}\right| = \lim_{n\to\infty} \frac{|x|}{(n+1)} = 0 \quad \text{for all } x.$$

So, $\sum_{n=0}^{\infty} u_n$ is convergent for all x.

So, $\underline{\underline{R = \infty}}$

[R=∞ should be clearly written]