7.5 Strategy of Integration:-

Strategies:

1. Simplify the integrand algebraically and see if it is integrable.

$$\int \frac{\tan\theta}{\sec^2\theta}\,d\theta = \int \frac{\sin\theta}{\cos\theta}\cos^2\theta\,d\theta$$

$$= \int \sin\theta\cos\theta\,d\theta = \frac{1}{2}\int \sin 2\theta\,d\theta$$

2. Use substitution (Easy-Sub) (U-sub)

$$\int \frac{x^2\,dx}{x^3-1} \qquad x^3-1=u,\quad 3x^2\,dx = du$$

$$=\frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-1| + C$$

3. Identify the form of the integrand; trigonometric functions, rational functions, integration by parts.

\# NOT Every continuous functions have elementary functions as a integral. $\int e^{x^2}dx$ $\int \frac{e^x}{x}dx$.

(2)

$\displaystyle\int \frac{\cosh(\tan^{-1}(2x))}{1+4x^2}\,dx$

Let, $u = \cancel{\cosh}\,(\tan^{-1}(2x))$

$du = \dfrac{2\,dx}{1+4x^2}$

$$= \int \cosh u \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\sinh(u) + C = \frac{1}{2}\sinh(\tan^{-1}(2x)) + C$$

$\displaystyle\int \frac{\cos x}{1-\sin x}\,dx$

$u = 1-\sin x$

$du = -\cos x\,dx$

$$= -\int \frac{du}{u} = -\ln|u| + C = -\ln|1-\sin x| + C$$

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$$\int \frac{t}{t^4+2}\,dt$$

$u = t^2$

$du = 2t\,dt$

$$= \frac{1}{2}\int \frac{du}{u^2+2}$$

$u = \sqrt{2}\tan\theta$

$du = \sqrt{2}\sec^2\theta\,d\theta$

$$= \frac{1}{2}\int \frac{\sqrt{2}\sec^2\theta\,d\theta}{2\sec^2\theta}$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{t^2}{\sqrt{2}}\right) + C$$

③

Cover-up rule

10 $\displaystyle\int_0^4 \frac{x-1}{x^2-4x-5}\,dx$

$$\frac{x-1}{x^2-4x-5} = \frac{x-1}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$$

(i) Multiply both sides by $(x-5)$ and put $x=5$:

$$\left.\frac{(x-1)}{(x+1)}\right|_{x=5} = A \Rightarrow A = \frac{4}{6} = \frac{2}{3}.$$

(ii) Multiply both sides by $(x+1)$ and put $x=-1$

$$\left.\frac{x-1}{x-5}\right|_{x=-1} = B \Rightarrow B = \frac{-2}{-6} = \frac{1}{3}.$$

So, $\displaystyle\int_0^4 \frac{x-1}{(x-5)(x+1)}\,dx = \frac{2}{3}\int_0^4 \frac{dx}{x-5} + \frac{1}{3}\int_0^4 \frac{dx}{x+1}$

$$= \frac{2}{3}\ln|x-5|\Big]_0^4 + \frac{1}{3}\ln|x+1|\Big]_0^4$$

$$= \frac{2}{3}\left[\ln 1 - \ln 5\right] + \frac{1}{3}\left[\ln 5 - \ln 1\right]$$

$$= -\frac{2}{3}\ln 5 + \frac{1}{3}\ln 5 = \underline{-\frac{1}{3}\ln 5}.$$

19. $\int e^{x+e^x}\,dx$

use, $e^{x+e^x} \Rightarrow e^x \cdot e^{e^x}$

$= \int e^x \cdot e^{e^x}\,dx$

let, $e^{e^x} = u$

$e^{e^x} \cdot e^x\,dx = du$

$= \int du = u + C$

$= e^{e^x} + C$

21. $\int \tan^{-1}(\sqrt{x})\,dx.$

$\sqrt{x} = u$

$\frac{1}{2\sqrt{x}}\,dx = du$

$= 2\int u \tan^{-1}(u)\,du$

$= 2 \tan^{-1}u \int u\,du - 2\int \frac{1}{1+u^2}\cdot\frac{u^2}{2}\,du$

$= u^2 \tan^{-1}u - \int \frac{u^2}{1+u^2}\,du$

$= u^2 \tan^{-1}u - \int du + \int \frac{du}{1+u^2}$

$= u^2 \tan^{-1}u - u + \tan^{-1}u + C. \quad = (u^2+1)\tan^{-1}u - u + C$

$= (x+1)\tan^{-1}(\sqrt{x}) - \sqrt{x} + C$

22. $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$

$1+(\ln x)^2 = u^2$

$\cancel{2}\ln x \cdot \frac{1}{x}\,dx = \cancel{2}u\,du$

$= \int \frac{u\,du}{u} = u + C = \sqrt{1+(\ln x)^2} + C$

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30 $$\int_{-1}^{2} |e^x - 1|\, dx$$

$$= \int_{-1}^{0} (1-e^x)\, dx + \int_{0}^{2} (e^x - 1)\, dx$$

$$= \left[x - e^x\right]_{-1}^{0} + \left[e^x - x\right]_{0}^{2} = \underline{e^2 + e^{-1} - 3}$$

39 Hint: Let, $\underline{\sec\theta = u}$

48 $$\int_{0}^{1} x\sqrt{2 - \sqrt{1-x^2}}\, dx \qquad 1 - x^2 = 4\sin^4\theta$$

$$-2x\,dx = 16\sin^3\theta\cos\theta\, d\theta$$

$$= -\int_{\pi/4}^{0} 8\sin^2\theta\cos\theta \cdot \sqrt{2}\cos\theta\, d\theta$$

$$= 8\sqrt{2}\int_{0}^{\pi/4} \sin^3\theta\cos^2\theta\, d\theta \qquad \cos\theta = v$$

$$-\sin\theta = dv.$$

$$= -8\sqrt{2}\int_{1}^{1/\sqrt{2}} (1-v^2)v^2\, dv.$$

$$= 8\sqrt{2}\left[\frac{v^3}{3} - \frac{v^5}{5}\right]_{1/\sqrt{2}}^{1} = 8\sqrt{2}\left[\frac{1}{3} - \frac{1}{5} - \frac{1}{6\sqrt{2}} + \frac{1}{20\sqrt{2}}\right]$$

$$= 8\sqrt{2}\left(\frac{2}{15} - \frac{7}{60\sqrt{2}}\right)$$

81 $$\int \sqrt{1-\sin x}\, dx$$

$$= \int \frac{\sqrt{1-\sin x}\,\sqrt{1+\sin x}}{\sqrt{1+\sin x}}\, dx = \int \frac{\cos x\, dx}{1+\sin x} \qquad \text{Let, } \sin x = u$$

$$\cos x\, dx = du$$

$$= \int \frac{du}{\sqrt{1+u}} = 2\sqrt{1+\sin x} + C.$$

$\int_0^1 \frac{x}{(x+1)^2}\,dx.$

$$\frac{x}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

① Multiply by $(x+1)^2$ and put $x=-1$

$$-1 = B$$

$$\frac{x}{(x+1)^2} + \frac{1}{(x+1)^2} = \frac{A}{x+1} \Rightarrow A=1$$

$$\int_0^1 \frac{x}{(x+1)^2}dx = \int_0^1 \frac{dx}{(x+1)} - \int_0^1 \frac{dx}{(x+1)^2}dx$$

$$= \ln|x+1|\Big]_0^1 + \left[\frac{1}{x+1}\right]_0^1$$

$$= \ln 2 + \frac{1}{2} - 1 = \underline{\underline{\ln 2 - \frac{1}{2}}}$$

61. $\int \frac{dx}{1+\cos x}$

$$= \int \frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}dx$$

$$= \int \frac{1-\cos x}{\sin^2 x}dx.$$

$$= \int \operatorname{cosec}^2 x\,dx - \int \frac{\cos x}{\sin^2 x}dx$$

$$= -\cot x + \operatorname{cosec} x + C$$

↑
by letting $\sin x = u$

62. $\int \frac{d\theta}{1+\cos^2\theta}$

$$= \int \frac{\sec^2\theta}{\sec^2\theta+1}d\theta = \int \frac{\sec^2\theta}{2+\tan^2\theta}d\theta$$

$$= \int \frac{du}{2+u^2}$$

$u = \tan\theta$
$du = \sec^2\theta\,d\theta$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan\theta\right) + C$$