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7.2

Trigonometric Integrals:-

1) $\int \sin^m x \cos^n x \, dx$.

→ If $n = 2k+1$ (odd), substitute $u = \sin x$.

$$\int \sin^2 x \cos x \, dx$$

$\sin x = u$

$+\cos x \, dx = du$

$$= +\int u^2 \, du$$

$$= +\frac{1}{3} u^3 + c = +\frac{1}{3} \sin^3 x + c$$

→ If $m = 2k+1$ (odd), Substitute $u = \cos x$.

$$\int \cos^2 x \sin^3 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$.

$$= \int (1-\cos^2 x) \cos^2 x \cdot \sin x \, dx$$

$$= -\int (1-u^2) u^2 \, du$$

$$= -\int (u^2 - u^4) \, du = -\frac{1}{3} u^3 + \frac{1}{5} u^5 + c$$

$$= -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + c$$

→ If both m & n are even, use double angled identity: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\sin 2x = 2\sin x \cos x.$$

$\frac{1}{2}(\sec^2 -1) + \frac{1}{4}(\sec \ldots)$

11> $\int \tan^m x \sec^n x \, dx$

→ If $n = 2k$ (even), substitute $u = \tan x$.

$\int \tan x \sec^4 x \, dx$ $\qquad u = \tan x$

$du = \sec^2 x \, dx$

$= \int u(1+u^2)\, du$

$= \frac{1}{2}u^2 + \frac{1}{4}u^4 + c = \frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + c$

→ If $m = 2k+1$ (odd), substitute $u = \sec x$.

$\int \tan x \sec^4 x \, dx$

$u = \sec x$

$= \int u^3 \, du$ $\qquad du = \sec x \tan x \, dx$

$= \frac{1}{4}u^4 + c = \frac{1}{4}\sec^4 x + c$

\# $\int \sec x \, dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$= \int \frac{du}{u}$ $\qquad u = \sec x + \tan x$

$du = \sec x(\tan x + \sec x)\, dx$

$= \ln|\sec x + \tan x| + c$

③

III> $\int \sin mx \cos nx \, dx$ or $\int \sin mx \sin nx \, dx$ or

$$\int \cos mx \cos nx \, dx.$$

In All instances use.

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

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$$\int \cos \pi x \cos 4\pi x \, dx$$

$$= \frac{1}{2}\int \left[\cos 5\pi x + \cos 3\pi x\right] dx$$

$$= \frac{1}{2}\,\frac{\sin 5\pi x}{5\pi} + \frac{1}{6\pi}\sin 3\pi x + C$$

49. $\int x \tan^2 x \, dx = \int x(\sec^2 x - 1)\, dx$

$$= \int x \sec^2 x \, dx - \frac{1}{2}x^2 + C$$

$$= x\tan x - \int \tan x \, dx - \frac{1}{2}x^2 + C$$

$$= x\tan x - \ln|\sec x| - \frac{1}{2}x^2 + C$$

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\# $\displaystyle\int_0^{\pi/4} \sin^3(12x)\cos^9(12x)\,dx.$

$u = \cos(12x)$

$du = -12\sin(12x)\,dx$

$$= -\frac{1}{12}\int_{1}^{-1} (1-u^2)\,u^9\,du$$

$$= \frac{1}{12}\int_{-1}^{1} (1-u^2)\,u^9\,du.$$

Integrand odd so, $= 0$.

\# $\displaystyle\int \frac{\sin^2 x}{\sqrt{1-\cos x}}\,dx.$

$$= \int \frac{\sin^2 x}{\sqrt{2\sin^2 x/2}}\,dx = 4\int \frac{\sin^2 x/2\,\cos^2 x/2}{\sqrt{2}\,\cancel{\sin x/2}}\,dx$$

$$= 2\sqrt{2}\int \sin x/2\,\cos^2 x/2\,dx$$

$u = \cos x/2$

$du = -\frac{1}{2}\sin x/2\,dx$

$$= -4\sqrt{2}\int u^2\,du$$

$$= -4\sqrt{2}\,\frac{1}{3}u^3 + c$$

$$= -\frac{4}{3}\sqrt{2}\,\cos^3 x/2 + c$$

45. $\displaystyle\int_0^{\pi/6} \sqrt{1+\cos 2x}\,dx = \int_0^{\pi/6} \sqrt{2}\cos x\,dx$

$$= \sqrt{2}\,\sin x\Big]_0^{\pi/6} dx = \sqrt{2}\,\frac{1}{2} = \frac{1}{\sqrt{2}}.$$