Work 5.4

Work = Total amount of effort to perform a task.

Newton's 2nd Law of Motion:

particle of mass m ⟵ F external force
moves along st. line s(t)

$$F = m\frac{d^2s}{dt^2} \quad N.$$

$W = F \cdot d$. (F is a cons't. force, d is the distance move)

↓ ↓

Newton . Meter (Joule)

or Pound . feet (1.36 J)

⇒ Difference between weight & mass. (Ex M 1), Work by const. force
mg m variable force

⇒ Varying force F.,

$$W = \int_a^b f(x)\,dx.$$ where, f(x) is the variable force.

↑

work in moving the object from a to b.

(See Ex 2)

Important: Should use unit at the end, J or ft-lb.

- Hooke's law: F to extend or compress a spring is proportional to the distance.

$f \propto x \Rightarrow F = kx$ k = spring const.

(Spring)

equilibrium or normal position

* Ex. Calculate work in stretching a spring from x = 2m to x = 5m. assuming initial f(2) = 1 N.

$\Rightarrow F(2) = K \cdot 2 = 1$

$\Rightarrow K = \frac{1}{2}$

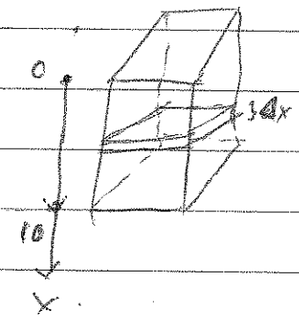
$F(x) = \frac{1}{2}x$.

$$W = \int_2^5 F(x)\,dx = \frac{x^2}{4}\Big]_2^5 = 5.25\ J.$$

Ex: (Problems related to gravity) (Ex. 5)

Underground tank of 6m × 6m wide and 10m deep; top at the ground level. How much work in pumping all the water up to ground level? (Water density 1000 Kg/m³)

Soln: Consider vertical co-ordinate system.



Volume of the water of the slice, dV

$= 6 \times 6 \times \Delta x$

$= 36\,\Delta x\ m^3$.

So, the mass, $m = dV \times$ density

$= 36\Delta x \cdot 1000 = 36{,}000\,\Delta x$ Kg.

The force required to raise the layer

= the weight of the water slice

$= 36000 \times 9.8\,\Delta x$ N,

Work $W = F \times x = 36000 \times 9.8 \times \Delta x$ J.

Work 5.4

Work = Total amount of effort to perform a task.

Newton's 2nd Law of Motion:

particle of mass m ← F external force

moves along st. line s(t)

$F = m \frac{d^2s}{dt^2}$.

Total work, $W = \int_0^{10} 36000 \times 9.8 \cdot x \, dx$

$= 352,800 \int_0^{10} x \, dx = \frac{352,800}{2} \times 10^2$

~~$= 176,400,000$~~

$= 1764 \times 10^6$ J.

5.4

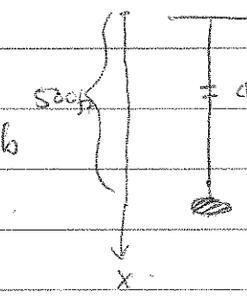
11.

Suppose, $f(x) = kx$. , force.

$W_1 = \int_0^{0.01} kx \, dx \qquad W_2 = \int_{0.01}^{0.02} kx \, dx$

$\Rightarrow \frac{W_2}{W_1} = 3.$

15.



~~Work~~

Const ~~force~~ $W_1 = 800 \times 500$ ft-lb

$= 400,000$ ft lb.

Variable work. weight of ith part $= 2\Delta x$.

Work $= 2\Delta x \cdot x$.

Total work $= \int_0^{500} 2x \, dx = (500)^2$ ft-lb.

Total work $= (500)^2 + 400,000 = 650,000$ ft lb.

1.

Work.

Const. force (Easy) (F.d)

Variable force (Hooke's law, gravity with cable weight)

$\int_a^b f(x) dx$