SCHOOL OF BASIC SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY BHUBANESWAR
Numerical Techniques Spring 2020

Problem Sheet-1

1. Suppose that m is a fixed positive integer. Show that the initial value problem
$$y' = y^{2m/(2m+1)}, \quad y(0) = 0,$$
has infinitely many continuously differentiable solutions. Why does this not contradict Picard-Lindelof Theorem?

2. Consider the following Runge-Kutta method for the IVP: $y' = f(x, y), y(x_0) = y_0$:
$$\begin{array}{c|cc} 0 & & \\ 1 & 1 & \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$
Show that the method has local truncation error
$$e_n = \frac{h^3}{6}\left[f_y(f_x + f_y f) - (f_{xx} + 2f_{xy}f + f_{yy}f^2)/2\right] + \mathcal{O}(h^4).$$

3. Construct a three-point finite difference scheme for the numerical solution of the boundary value problem
$$y'' + x^2 y = 0, \qquad y(0) = 1, y'(1) = 0,$$
on the interval $[0, 1]$. Show that the resulting system of equations can be written so that the matrix of the system becomes tridiagonal.

4. Using the Runge-Kutta method of order 2, solve
$$Y'(t) = Y(t) + t^{0.1}(1.1 + t), \quad Y(0) = 0,$$
whose solution is $Y(t) = t^{1.1}$. Solve the equation on $[0, 5]$, with 51 partition points and find the approximate value of $Y(0.2)$.

5. Consider the BVP
$$Y''(t) = 0, \quad Y'(0) = g_1, Y'(1) = g_2.$$
Show that the problem has no solution if $g_1 \neq g_2$, and infinite solutions when $g_1 = g_2$.

6. Write the linear system correcponding to the BVP
$$tY'' - (2t + 1)Y' + (t + 1)Y = 0, Y'(1) = 0, Y(2) = e^2.$$
Write down a formula for a discrete approximation of the boundary condition $Y'(1) = 0$, which has an accuracy $\mathcal{O}(h^2)$.

7. Consider the IVP
$$x'' + 4x' + 5x = 0, \quad x(0) = 3, x'(0) = -5.$$
Write down the equivalent system of 1st order equations. How do you use Euler method to solve the system the reformulated problem over $[0, 5]$ using $N = 50$ subintervals.

8. Solve the IVP $x' = 2x^2 t, x(0) = 1$ at $t = 0.2$ using Picard and Euler method.
9. Apply RK4 to find the solution of $y' = 3x + y/2$ with $y(0) = 1$ at $x = 0.1$.

10. Apply the following scheme to find the solution of $y' = 3x + y/2$ with $y(0) = 1$ at $x = 0.1$:

$$\begin{array}{c|cc} 0 & & \\ 2/3 & 2/3 & \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array}$$