SCHOOL OF BASIC SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY BHUBANESWAR
Numerical Techniques (MA2L007) Spring 2020

Problem Sheet-8 (Miscellaneous)

1. Show that both the Jacobi and Gauss-Seidel methods are convergent when the matrix has a strictly dominant diagonal.

2. Discuss the convergence of the Jacobi and Gauss-Seidel processes for the solution of $Ax = b$ when
$$A = \begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & -1 \\ -2 & -2 & 1 \end{pmatrix} \text{ and } \quad A = \begin{pmatrix} 1 & 1/2 & -1/2 \\ -1 & 1 & -1 \\ 1/2 & 1/2 & 1 \end{pmatrix}.$$

3. Find the equation of the parabola which coincides with the sine curve $y = \sin x$ at $x = 0, (\pi/2), \pi$.

4. Find the larger root of the equation $f(x) = x^2 - 3.6 \log_{10} x - 2.7 = 0$, correct to four decimal places (assume 3.6 and 2.7 are exact numbers).

5. Use the two-dimensional Newton-Raphson Method to solve the two simultaneous non-linear equations
$$x^2 + (y-4)^2 - 9 = 0,$$
$$(x-4)^2 + y^2 - 9 = 0,$$
using the initial values $x_0 = 2.4$ and $y_0 = 2.6$.

6. A body of mass 2 Kg is attached to a spring with a spring constant of 10. The differential equation governing the displacement of the body y and time t is given by: $y'' + 2y' + 5y = 0$. Find the displacement y at time $t = 0.5, 1, 1.5$ using finite difference method. Given that $y(0) = 2, y(2) = 4$.

7. Solve the Poisson's Equation $\nabla^2 f = 2x^2y^2$ over the square domain $[0,3] \times [0,3]$ with $f = 0$ on the boundary and $h = 1$.

8. Estimate the values at grid points of the following equation using Bender-Schmidt Recurrence Equation. Assume $\Delta t = \Delta x = 1$.
$$0.5 f_{xx} - f_t = 0,$$
$$f(0,t) = -5, f(5,t) = 5$$
$$f(x,0) = -5, \text{for } x : [0, 2.5]$$
$$= 5, \text{for } x : [2.5, 5]$$

9. (a) If $f(h) = o(g(h))$, then show that $f(h) = \mathcal{O}(g(h))$.
(b) Give an example to show that the converse is not true.
(c) What is meant by $f(h) = o(1)$ and $f(h) = \mathcal{O}(1)$?
(d) Give an example of $f(h)$ and $g(h)$ such that $f(h)$ is much bigger than $g(h)$, but still $f(h) = \mathcal{O}(g(h))$ as $h \to 0$.

10. Show that the matrix
$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}.$$
is invertible but has no LU factorization. Do a suitable interchange of rows and/or columns to get an invertible matrix, which has LU factorization.

11. Give an initial guess x_0 for which the Newton-Raphson method fails to obtain the real root for the equation $\frac{1}{3}x^3 - x^2 + x + 1 = 0$. Give reason for why it failed.

12. Use Newton's method to find the minimum value of the function $f(x_1, x_2) = x_1^4 + x_1 x_2 + (1 + x_2)^2$.

13. Show that the Euler and Runge-Kutta methods fails to determine an approximation to the non-trivial solution of the initial-value problem $y' = y^a, a < 1, y(0) = 0$, although the exact (non-trivial) solution exists.
14. Regard the equation $e^x + ax + b = 0$. Choose a, b so that the equation has exactly one negative solution. Then find this solution using Newton's method.