I.A.Sc Research Fellowship-2009

Final Report of the Fellowship

- Name of the Summer Fellow:- **Bankim Chandra Mandal**
- Name of the Supervisor/Guide:- **T.E.Venkata Balaji**
- Date of Commencement of the Fellowship:- $30^{th} April, 2009$

§ Brief Report of Topics Studied :-

As I mentioned in my first progress report, my area of interest is Complex Analysis. On the first day of my joining, my guide gave me some idea about Riemann Surfaces. He explained the definition and indicated how the Riemann Sphere is a Riemann Surface.I have worked out various examples of Riemann Surfaces and have found the exact expressions for the transition functions.I have learnt about the notions of analytic and meromorphic functions using the corresponding notions on the complex plane. For a general Riemann Surface R, I have studied the notions of $\Theta(R)$,(set of all analytic maps $f : R \to \mathbb{C}$), which is a vector space and $\mu(R)$,(set of all analytic maps $f : R \to \mathbb{C}_\infty$), which is a field, namely the function field of R.I have read about fundamental group ($\Pi_1(R)$), Homology groups, Covering Manifolds, Ramification number of a holomorphic map at some point and the notion of open & closed Riemann Surfaces.

In this course of time, I also studied about Triangulation of a compact surface, genus, complex tori and the classification of Riemann Surfaces. I have gone through the concepts of Homotopy, Fundamental group, Covering spaces & Homology in Algebraic Topology for understanding some related things in my topic.

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✠ I have used the following books as references :

1) Riemann Surfaces (by H.M.Farkas & I.Kra)

2) Riemann Surfaces (by Pablo Arés Gastesi)

3) Complex Analysis (by R.Narasimhan)

4) Complex Analysis (by Gamelin)

5) General Topology (by J.Munkres)

✠ In this connection, I would like to give some important results, that I have studied:

- [**Principle of Analytic Continuation**] :-

  Let, X & Y be Riemann Surfaces and let, $f : X \to Y$ be analytic. If there is a non-empty open set U_0 of X such that $f|_{U_0}$ is a constant y_0, then $f \equiv y_0$.

- [**Open Mapping Theorem**] :-

  Let, X & Y be Riemann Surfaces and let, $f : X \to Y$ be analytic.If f is not constant, f is an open map.

- [**The Maximum Principle**] :-

  Let, X be a Riemann Surface & $f \in \Theta(X)$.If $\exists$ a $\in X$ such that $|f(x)| \leq |f(a)|$, $\forall x \in X$; then f is a constant function.

- [**Weierstrass' Theorem**] :-

  Let, X be a Riemann Surface and $\{f_n\}_n$ be a sequence of functions in $\Theta(X)$. If $\{f_n\}_n$ converges uniformly on compact subsets of X, then the limit is also analytic on X.

- [**Montel's Theorem**] :-

  Let, x be a Riemann Surface and let $F \subseteq \Theta(X)$ be a subset such that $\sup\limits_{f \in F} \sup\limits_{x \in K} |f(x)| < \infty$ for any compact set $K \subseteq X$. Then, any sequence $\{f_n\}_n$ of function in F has a subsequence converging uniformly on compact subsets of X.

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- [**Riemann Extension Theorem**] **:-**

  Let, X be a Riemann Surface and let, $a \in \Theta(X - \{a\})$ be such that, $\exists$ a neighbourhood U of a for which $f|_{U-\{a\}}$ is bounded. Then, $\exists\, F \in \Theta(X)$ with $F|_{X-\{a\}} = f$.

- [**Uniformization Theorem for Riemann Surfaces**] **:-**

  Each simply connected Riemann Surface is conformally equivalent to either the open unit disc ($\triangle$), the complex plane ($\mathbb{C}$) or the Riemann Sphere ($\mathbb{C} \cup \{\infty\}$).

- The set of equivalence classes of tori is in $1-1$ canonical correspondence with the points in $\mathbb{C}$.

✠ I have gone through the topics on some exceptional Riemann Surfaces & prolongable Rieman Surfaces.
Lastly,I am very grateful to my guide for his helpful guidance. It was a very pleasant learning experience for me and I am sure that,it will help me a lot in my higher studies. I would like to continue my work with my guide later on, if I get sufficient scope. Also, I would like to give thanks to Dr.Swadesh Sahoo for his whole-hearted help.

§ **Other Academic Activities :-**

✓ I have learnt LaTeX, which will be helpful for me later on.

✓ I have given a talk on "Introduction to Fundamental group, Homotopy & Covering Spaces in Algebraic Topology at IIT Madras.Some of the professors and all summer fellows were present there.

✓ I have attended the following lecture series for summer students at Chennai Mathematical Institute, being held from 14^{th} May to 25^{th} June :

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Lecturer	**Topic**	**Date**
Amritanshu Prasad	Finite Abelian Group	14^{th} May
S.P.Suresh	Introduction to Logic	14^{th} May
Murali K.Vemuri	Uniform Convergence	18^{th} May
N.Saradha	Irreducibility of polynomials via Newton's Polygon	20^{th} May
Anirban Mukhopadhyay	Quadratic Reciprocity	21^{st} May
Anand Deopurkar	Counting Problems in Geometry	21^{st} May
Purusottam Rath	Prime Numbers	25^{th} May
V.Balaji	Group Action	28^{th} May
M.Sundari	Fourier Series	28^{th} May
Shrihari Sridharan	Arbitarily Long Arithmetic Prograssions	1^{st} June
S.Kesavan	Classical Isoperimetric Inequality	4^{th} June
Krishnan Rajkumar	Irratioanal Numbers	4^{th} June
K.V.Subrahmanyam	Discrete Fourier Transforms	8^{th} June
P.Venchinathan	Finite groups & Symmetries-1	11^{th} June
S.S. Kannan	Finite groups & Symmetries-2	11^{th} June
V.Uma	Differential Manifolds	15^{th} June
R.Radha	Sequence of Functions	18^{th} June
Suresh Nayak	Bezout's Theorem	22^{nd} June
V.V.Sreedhar	Knot Theory	25^{th} June

Name of the Summer Fellow ______________________   Name of the Guide ______________________

Signature of the Summer Fellow ______________________   Signature of the Guide ______________________

Date: __________   Date: __________